Among the most frequently used words in English are simple prepositions like *in* or *on* whose basic meaning is spatial in character. What we propose in this article is a classification of the kinds of geometry that underlie the basic meaning of various spatial prepositions. The distinctions among geometries underlying these prepositions has not, as far as we can tell, been previously noted in the literature of linguistics and philosophy. In fact, in spite of the spate of articles in the last decade or so on locative expressions, spatial prepositions, and the like, detailed attention to the kinds of geometry needed to give a semantic analysis of the various locative expressions does not seem to have been previously attempted.

For example, consider the work of Cresswell (1978) which gives an interesting analysis of points of view, meaning spatial points of view, that are implicit in understanding the meaning of such words as *come*, *go*, *left*, *right*, and *behind*, to use his initial list. Cresswell goes far beyond what we attempt here in that he gives a formal semantics in terms of the categorical semantics he has written about extensively. But while we do not formalize the semantics we outline, in contrast with Cresswell we distinguish between spatial prepositions requiring a point of view and those that do not and we place this distinction, along with others, in a geometrical framework. For example, if someone says *The book you want is to the left of the dictionary* we all immediately understand that a point of view is involved. On the other hand, we are just as firmly convinced, without argument, that in many uses of language we describe the world in terms of the relations between objects and possibly other phenomena, without any attention to an implicit point of view on the part of the speaker or writer. It would seem absurd in most contexts to say that the pencil is on the table from my point of view but not from yours. Thus the ordinary primitive or basic meaning of *on* is in terms of a geometry that does not assume a point of view. However, the preposition *on* assumes a physical ori-
entation of verticality that is missing, for example, from three-dimensional Euclidean geometry. We can rightly claim that in many contexts the basic or primitive meaning of the preposition in does not assume any such orientation and can be given a perfectly reasonable account in terms of a weaker geometry, as when a child says The penny is in the piggy bank.

Still other kinds of geometrical considerations enter when processes or events are talked about. For example, when we talk about the location of processes it is appropriate to think of the underlying geometry as being four-dimensional space-time, as we shall illustrate later. Perhaps the most interesting point is that these different underlying geometries used to formulate the appropriate geometrical semantics for spatial prepositions reflect the different ways in which we talk about the world in a spatial sense.

We emphasize that in no sense do we think that purely geometrical analysis can provide a satisfactory account of the meaning of spatial prepositions. The uses are too many and varied and the ways in which language is used too flexible to sanction any simple geometrical account as being wholly adequate. There are at least three major reservations that need to be made. First of all, even prepositions that seem mainly spatial depend also on additional assumptions about the physical world. For example, in the use of the common preposition on there is a familiar notion of support that is physical rather than geometrical in character. In the next section we characterize on in purely geometrical terms but this does not mean we think that characterization is complete. We only mean to catch in the definition given the geometrical aspects of the meaning.

Second, even if we bring in commonsense physics, the context of use is an even more important aspect of meaning that prohibits any simple account of the semantics of the prepositions we consider. In other publications we have emphasized, from our standpoint, the importance of context in fixing the meaning of words, phrases, and sentences (Crangle and Suppes 1987). Everyone who has thought about these matters is aware of the importance of context, so that we shall not say more on this point until our later detailed discussion of several examples. The brevity of what we say here is not in any sense meant to decrease the essential and salient role of context in fixing the meaning of locative expressions.

Third, even the purely geometrical interpretation is not straightforward. The geometry of ordinary objects and of talk about these objects is not the pristine and precise geometry of Euclid and his mathematical successors. What we have to say in the next section builds upon Euclid but this is just because we do not have an adequate alternative foundation. There is in the use of spatial language an inherent vagueness intrinsic to the geometry itself that we will not characterize satisfactorily in a formal way. When someone says The pencil is near the book we are not able to define a precise geometrical semantics to near that will deliver the appropriate truth conditions. Even when we take in the context of objects, we will still be faced with difficulties in saying what the exact meaning is. This is just one of many different kinds of exam-
pies. It is important to emphasize this class of examples, however, because they show that even within the framework of purely geometrical considerations we cannot hope to achieve the kind of result familiar in the formal analysis of the semantics of geometrical theories in the standard mathematical sense.

These many different reservations do not mean we consider it impossible to make any distinctions. Our insistence on different types of geometry underlying the basic meaning of different spatial prepositions is evidence to the contrary. However, there is one general philosophical remark we would like to make about the situation. Everyone who writes about language in any detail rapidly becomes impressed with the bewildering complexity and subtlety of actual language use. What we want to emphasize is that this is not a peculiar characteristic of language but is characteristic of all natural phenomena. The simplest sorts of physical phenomena exhibit the same kind of bewildering complexity and subtlety. We think that our understanding of physics is better than our understanding of language, from the standpoint of systematic analysis, more because of the successful isolation over a long period of time of a few salient cases that are simple: two bodies in planetary motion, a free-falling body, the pendulum of a clock, and so forth. But if we just walk out and look at some physical phenomenon occurring as a breeze sweeps across a patch of dust and leaves, we are no more able to account for the details of the motion of the objects than we are of the kinds of linguistic examples we can as easily generate. Of course, physics is a more developed science than linguistics, and the philosophy of physics reflects this long development in comparison with the philosophy of language. On the other hand, we think that the similarities regarding problems of complexity are much more important than the differences. The case of language is not at all unique but is typical for the analysis of any natural phenomenon. We can no more give a really satisfactory foundational account of physical phenomena than we can of linguistic phenomena. What we hope to do in both cases is to be lucky enough to find some salient distinctions in ways of looking at the phenomena that advance our understanding a modest amount. It is this viewpoint that has characterized what we have tried to do in this article.

Current work on the spatial locatives is characterized in large part by the recognition that both geometrical and physical notions are needed for an analysis of the locatives, but without any detailed exposition of those notions being given. Bennett (1975), for instance, provides a componential analysis in which the components (or "semantic markers" as they are sometimes called) include the concepts of interior, surface, anterior (the space in front of something), and posterior (the space behind something). For example, the concept of interior is thought to be part of the meaning of the prepositions *in*, *into*, and *through* but not, say, *behind*. Perhaps precisely because Bennett's emphasis is the relationships holding between words, not the relationship, as he puts it, between words and the world, he provides no analysis of any of the geometrical notions that he uses. Components or concepts such as interior and
surface function as unanalyzed wholes. These he uses along with notions such as source, path, and goal supplied by his framework of case grammars to explicate various semantic notions. In our view, this level of abstraction from geometrical details precludes adequate accounts of the locatives in English. Their meaning is too intricately tied up with geometrical and physical facts about the world. Bennett's own brief discussion of the following two sentences supports this view even though he fails to incorporate such facts in his analysis. Consider the sentences (a) Judy fell over the cliff and (b) Judy fell over the curb. The word over works differently in the two sentences. An informal paraphrase of the first is Judy fell downwards from the edge of the cliff, a reading that results because we take Judy to be considerably smaller than the cliff. A paraphrase of the second is Judy fell as a result of a collision with the curb, a reading that results because we take Judy to be somewhat larger than the curb. However, if 'Judy' names a giant in (a) and a creature the size of an ant in (b), the readings are reversed.

A more recent study of the prepositions in English is found in Herskovits (1986). Herskovits proposes various geometric idealizations, and adaptations of these idealizations, that are thought to underlie the many different uses of a preposition. Her work is worth particular mention here in that it acknowledges the extent to which context intrudes into the interpretation of a locative expression. However, like Bennett, Herskovits does not make explicit the various geometrical notions such as interior, outline, contiguous, and at-the-back that appear in her analysis. They are consequently left unanalyzed and as a result no unifying geometrical or computational framework is provided.

A work that must be mentioned in relation to ours is that of Talmy (1983). Talmy recognizes many of the basic geometrical distinctions we do but fails to organize these distinctions along their natural geometrical lines. Instead he proposes four "imaging systems" a speaker is thought to use in forming spatial expressions and he proposes various spatial schemata the expressions are thought to represent. These schemata are said to be built up from rudimentary spatial elements such as points, lines, and planes. We find ourselves in serious disagreement with Talmy's approach and trace this disagreement to the absence of a coherent geometrical framework in his work. This absence, we believe, is responsible for contradictions we note in his work. Talmy explicitly rejects the properties of metric spaces, claiming rather that a central role be assigned to qualitative or topological concepts in the analysis of spatial expressions. But his own careful discussion of specific prepositions is replete with references to properties such as length and breadth, and relations such as parallelism and longer than, all of which require at least an affine not merely a topological space. In addition, it is abundantly clear that we usually need a metric for any adequate analysis of spatial expressions. We will in fact show in this article that, far from eliminating Euclidean geometry as Talmy seems to suggest, a more productive strategy is to strengthen it in various ways. One of the ways in which it should be strengthened is with a theory of geometrical figures so that we do not always have to take as primitives points in space but
can rather express basic spatial relations in terms of figures and their parts. This possibility seems to be what Talmy is driving at when he talks about abstracting from specifics such as the magnitude of points, lines, and planes and focusing rather on topological properties such as distinguishability of parts or "partiteness." Talmy's work, rich as it is in subtle examples, shows more clearly than any other the need for an organizing classification of the geometry that underlies spatial expressions. We make a start on this task in this article.

SOME RELEVANT GEOMETRIES

We have entitled our semantic analysis of spatial locatives geometrical semantics to emphasize that we use throughout in the set of models admitted strong geometrical assumptions that go beyond ordinary set-theoretical semantics. It is our conviction that in the use of language to describe events, actions, or situations in space and time the largest piece of unfinished business is the spelling out of the geometrical and physical assumptions that are implicit in the semantics of ordinary talk. There is an implicit mistaken view in much general semantic analysis by philosophers and linguists that it is adequate to give a purely set-theoretical framework that does not go beyond the axioms of general set theory. In our view, in contrast, the real task is to analyze the implicit assumptions of a spatial or temporal nature.

Much work must be done before a satisfactory theory of geometry and physics implicit in ordinary language is put in a satisfactory form. What we do in this article is modest in scope, but even with this limitation some geometrical distinctions that seem essential to distinguishing among spatial locatives can be made. Roughly speaking, the idea of our classification is to begin with affine geometry, to which the concept of parallel lines is central, but for which there is no concept of perpendicular lines or a natural metric, and then to strengthen it in various ways to give a satisfactory account of the implicit geometrical semantics behind the understood meaning of a particular spatial locative. In brief terms, some locatives can be given a reasonably satisfactory semantical underpinning just in terms of affine or Euclidean space. Others require that Euclidean geometry be strengthened to an oriented geometry with a preferred vertical direction. Notice that in most ordinary talk we have an agreed-upon direction of vertical up and down, but no preferred horizontal direction. The horizontal direction is fixed by the introduction of restricted projective geometry and the concept of a station point for a perspective point of view. We restrict the projective geometry to the projective geometry of the plane because most of the points of view in the use of spatial locatives can be so restricted. We also consider the geometry that is intrinsic to individual bodies in terms of an orientation. For example, an object may have at least one axis of symmetry, and also in terms of use a designated front and back, top and bottom, and so on. We end our proposed classification by considering some prepositions that are topologically invariant.
In our discussion of these geometrical distinctions and concepts, we will take an informal route but provide enough technical detail to make clear how a more formal development can be given. There is one aspect of the geometrical developments that is not really satisfactory. Just because of its familiar and well-developed character we have begun with the standard conception of geometry in terms of a basic set of points. It is clear that the geometry underlying ordinary language is intrinsically of a different character, being deeply intermixed with physics. The natural primitive elements are not points but physical objects, as well as fluids and gases. There is not a well-developed foundation in these terms and we are not able to begin where we would really like to. It is a matter for research not yet undertaken to provide a foundation of geometry more closely matching the use of concepts in ordinary language. (For review of the earlier literature, see Suppes 1972).

**Affine geometry.** We shall consider affine geometry as given by the usual primitives of a set $A$ of points and the ternary relation $B$ of betweenness between points. (This is actually ordered affine geometry.) For a recent development, see Suppes et al. (in press). We shall assume all the standard geometrical relations that can be developed that are familiar for affine geometry. As already noted, the key concept is that of two lines being parallel. A good example of a spatial-locative for which a fairly good account can be given just in terms of affine geometry is the preposition *in*. Thus, for the analysis of the sentence *The pencil is in the box* where $P$ is the set of points constituting the pencil and $B$ the set of points constituting the box, one might give as a simple set-theoretical analysis that $P$ is a subset of $B$. Of course this is a mistake because the box itself does not include the inside open space. We are especially thinking here, as an example, of a box without a top. On the other hand, the right geometrical notion is to take the convex closure of $B$, that is, the set of all points lying on any segment connecting any two points in $B$. Then, when we say that the pencil is in the box we can express this geometrically by saying that $P$ is contained in the convex closure of $B$. Notice that orientation enters into our judgment of whether or not the pencil will stay in the box, as we move the box about, but no concept of orientation is needed for our judgment of whether or not the pencil is in the box.

We also note immediately that for many cases of usage some adjustment to this condition has to be made. Consider, for example, the sentence *The flowers are in the vase*. As ordinarily understood, the flowers are not in the convex closure of the vase but only the lower part of the stems of the flowers is in the convex closure of the vase. A similar example is *The bat is in his hand*. In a thorough and detailed analysis we would need to separate those cases of *in* which are covered in a straightforward way by the convex closure of the container from those that require more subtle constraints.

A second obvious example suitable for affine geometry is just the preposition *between*, based as it is on the primitive relation of betweenness. The sentence *Mary is sitting between José and Maria* does not imply exact linearity
of seating arrangement, but would ordinarily be assumed to be approximately so.

*Euclidean geometry.* The best-known modern axiomatization of Euclidean geometry, at least among philosophers, is that of Tarski (1959). To the primitives of affine geometry he adds the standard relation $\equiv$ of congruence for line segments—technically formulated for quadruples of points. An example fitting clearly within Euclidean, and not merely affine, geometry is the spatial locative *near.* Thus, consider the sentence *The pencil is near the box.* We can introduce a metric $d$ for Euclidean distance, and there are of course here several alternatives. It seems to us there is no single metrical relation that expresses nearness but several that may be considered satisfactory. The simplest in concept we feel is one that says simply that the minimum distance between the box and the pencil is less than some $\epsilon$, say $\epsilon_1$, which at the least is a function of the size of the two objects. We may express this relation as follows:

$$0 < \min_{p \in P, b \in B} d(p, b) < \epsilon_1(P, B).$$

A very different $\epsilon$ is needed to express nearness when the objects are quite different in size, as in *Philadelphia is near New York, but Pittsburgh is not.* It is worth noting that it is not clear that we want the negation of nearness to be simply the change in inequality. We might want to think of it in terms of something stronger, that is, having a second $\epsilon, \epsilon_2$ that is greater than $\epsilon_1$. Thus, in that case we would formulate the underlying relation for the sentence *The pencil is not near the box* by the metrical relation:

$$\min_{p \in P, b \in B} d(p, b) \geq \epsilon_2(P, B).$$

It is even doubtful that a deterministic relation should be used. There is a long tradition in psychology, supported by a variety of empirical studies, that such threshold phenomena, exemplified by the meaning of *nearness,* should be represented by a probability distribution. Following this lead we would need to replace the metric $d(p, b)$ by a probabilistic metric.

*The geometry of oriented physical space.* To give a geometrical semantics for such simple spatial locatives as *on,* we need to move from Euclidean space to physical space, where what we mean by physical space in the present context is something quite restricted, namely, the addition of a preferred direction *up* and its converse *down.* Obviously, in ordinary experience the sense of up comes from gravitational pull. We can introduce it purely geometrically by introducing two distinguished points, $\alpha$ and $\beta,$ with the understanding that $\alpha$ is vertically above $\beta.$ Our restricted physical space is then based upon the quintuple $(A, B, \equiv, \alpha, \beta).$ We are then in a position to define various standard intuitive notions of vertical and horizontal: A line or line segment is *vertical* if and only if it is parallel to $\alpha \beta.$ Correspondingly, a line or line segment is *horizontal* if and only if it is perpendicular to a line that is parallel to $\alpha \beta.$ (Note that in these definitions when we talk about a segment being perpendicular we of course have in mind the extension of the segment to the full line.) Next we can define a Euclidean plane as horizontal if and only if it is
perpendicular to $\alpha \beta$. In this restricted physical space we have the obvious theorem: *Any point lies on a unique horizontal plane.* For a given point $a$ we designate the unique horizontal plane $H(a)$. Notice that without introducing a point of view, i.e., station point, we have no natural orientation in a horizontal plane. We have in restricted physical space just the natural orientation of up and down or, more abstractly, the natural orientation of verticality. Using the concepts introduced, we can characterize the spatial relation *higher than.* Point $a$ is higher than point $b$ if and only if $a = \alpha$ and $b = \beta$, or either segment $a\alpha$ does not intersect $H(b)$ or segment $b\beta$ does not intersect $H(a)$. We can now define the spatial locative *above* for points. Point $a$ is above point $b$ if and only if $a$ is higher than $b$ and the segment $ab$ is approximately vertical. It is apparent that we can replace *approximately* by a metrical formulation in terms of a small angular disparity from the vertical, but no real gain in clarity would result. We use this same concept of approximate in defining *above* for bodies. There are three natural cases. Body $A$ is above body $B$ if and only if

(i) If $A$ and $B$ have approximately the same horizontal profile, then every point of $B$ has some point of $A$ above it,

(ii) if $A$ has a larger horizontal profile than $B$, then every point of $B$ has some point of $A$ above it,

(iii) if $A$ has a smaller horizontal profile than $B$, then every point of $A$ is above some point of $B$.

Using now *above* we can define the simple geometrical sense of the spatial locative *on*.

$A$ is on $B$ if and only if (i) $A$ and $B$ have a boundary in common and (ii) every interior element of $A$ is above some interior element of $B$.

*On* is an important case about which we need to say more. It is so widely used that it has many subtle aspects. For example, if a book is on the table in the sense just defined, and a second book is on this book in the sense just defined, we still are inclined to say that the second book is on the table, even though the definition just given is not satisfied because the second book and the table do not have a boundary in common. One way to solve this is to recognize that *on* as defined is ordinarily intransitive and to extend *on* to the relative product about four or five times. The number of elements is not very definite, but in most cases we would not use *on* when the number of intermediate elements exceeded four or five. Moreover, there are other still more subtle matters. The table is on the floor, we can all agree, but we are not inclined to apply transitive closure to say, therefore, that the book that is on the table is also on the floor. What blocks this simple closure where the same number of objects are involved, as in the case of saying that the second book on top of the first book is also on the table? We think that the main "change of pace" in concepts involved as we move from the table to the floor is that the boundary contact between the table and the floor is very limited, as opposed to the extensive flat contact between the book and the table. So this very limited
contact breaks the use of the transitive closure. Another different kind of consideration is the relative size of the table and the book. We ordinarily apply *on* as the relative product of several elements only until we reach a sizable stable object like a table. We do not go further. It would be a Lewis Carroll-type joke to go on and say, *Indeed the table is on the floor but the floor is on the ground.* By this point we get something that seems very counterintuitive. We do think that the relative size of the objects is as important as the minimal boundary contact between the table and the floor. We do not attempt a systematic analysis of these features in this article.

*Projective geometry.* We do not begin from scratch, as we might, in developing projective geometry, for in ordinary language and ordinary experience we commingle completely three-dimensional physical space and a perspective point of view. The topic of perspective itself is complicated and has a long history. We assume for this discussion the standard geometrical theory of perspective. A distinction is often introduced between "artist's perspective" and "geometrical perspective." In the former case, the artist attempts to draw the object as he sees it projected on the spherical surface of the retina of his eye, while geometrical perspective is very much what we see in a photograph, that is, projection onto a plane. Fortunately, except in wide angles of vision, the difference between the two is scarcely noticeable. What we have termed "artist's perspective" can also be given a geometrical treatment, but it is more complicated than that of projection onto a plane. Also, as already mentioned, for simplicity of formulation and because it covers most standard cases, we assume that the perspective point of view operates only in a horizontal plane, and not in any inclined plane. It is a standard psychological experiment, for example, in studying visual perception to have the viewer assume a horizontal position and to view perspective in the vertical plane. But this is not standard in most of ordinary experience, so that the restriction we impose is a natural one even if in no sense it is meant to cover all the cases that do occur. (For a good psychological analysis of the perception of vertical and horizontal distances in ordinary outdoor settings, see Higashiyama and Ueyama 1988.)

In the standard language of perspective the viewer is said to be at a *station point* and the projection takes place from that point. The station point is also often called the *point of view.* A fascinating thing about spatial locatives is that many which we think of initially in terms of three-dimensional physical space have a second interpretation in terms of relative position in the horizontal plane. For instance, in the case of *above* we have the following definition that covers the usage we have encountered in high school students' descriptions of place settings at a dinner table: *A is above B* (in the horizontal plane) if and only if *A* is further, in the same direction, from the station point than *B.* Two examples from our writing samples are *the cup above the plate* and *the glass above the knife.* In the same spirit we can characterize the locative *in front of:* *A is in front of B* if and only if *A* is closer, in the same direction, to the station point than *B.* Among the familiar spatial locatives
perhaps the ones most purely perspectival in nature are to the left of and to the right of, which need no further definition. But even in these apparently simple cases, complications lie close at hand. We ordinarily think of to the left of as denoting an ordering relation among objects seen from a particular point of view. However, when the occasion demands, we make a substantive out of the leftmost part of the field of view, as in Hand me the book on the left. The sense of spatial order from a point of view is still present but now not necessarily among given objects.

Geometries that include figures and shapes with orienting axes. Even when bodies are located in three-dimensional space with no sense of direction intrinsic to the space we still assign orienting axes and other geometrical distinctions to them. It is important to recognize that these distinctions are intrinsic to the body and remain with the body no matter what the body’s overall orientation is in physical space. In ordinary talk, for example, the top of the table remains the top of the table even if the table is sitting upside down on the floor. There is a whole language about this intrinsic orientation of bodies and there are also many cases of spatial locatives that involve the use of a body’s intrinsic orientation. For example, the instruction Stand in front of the house will in many instances be understood to refer to the side of the house where the main entrance is, the side that typically faces the street. This understanding of in front of would prevail no matter where the participants in the conversation were located. The object’s intrinsic orientation predominates. There are times when such an understanding of in front of may be overridden, however, especially when—unlike the above example—the reference object is smaller than the located object. In these cases a prevailing point of view may predominate. Consider the expression in front of the chair. A chair typically has an intrinsic front and back, yet despite this the command Stand in front of the chair will generally receive two competing interpretations, one in which the person stands at the chair’s own front, the other in which he or she stands on this side of the chair relative to some station point. (For a good analysis of how complicated the geometry of a chair is, see Strang 1982).

Geometry of classical space-time. For verbs of process or action, or for prepositional phrases locating events, it is often necessary to go beyond static three-dimensional physical space to four-dimensional space-time to provide the proper setting for their semantic analysis. Corresponding to oriented physical space, we use a four-dimensional affine space, and the sets of points simultaneous with any point is a three-dimensional physical space in the sense defined above.

It will perhaps be helpful to be more explicit about the primitive concepts. The four-dimensional space \((A,B)\), where \(B\) is the ternary relation of betweenness is affine. Note that the four-dimensional space-time is not Euclidean, for in classical space-time there is no comparison possible of the spatial distance between two points and the temporal distance between them. But the classical space-time is affine, i.e., the notion of being parallel is meaningful as well as the notion of linearity, both of which are definable in terms of between-
ness. Now let \( a \) be any space-time point. Then \([a]\) is the set of all points simultaneous with \( a \). For each point \( a \) the subspace \(([a], B, \approx, \alpha, \beta)\) is a three-dimensional oriented physical space, as defined above.

Let us now look at some uses of spatial prepositions for which such space-time geometry is appropriate. *She was hurt in an auto accident.* This sentence implies a clear sense of spatial location, but what is also essential is the sequence of events occurring there. In many other uses of *in* with verbs of process or action, the prepositional phrase locates the action, even if a strict sense of containment between objects does not hold. *She peeled apples in the kitchen. He walked in the park. She wrote her essay in her bedroom.* In these three sentences we can give a strict geometrical sense of location for the process, actions, or events, but our original geometrical scheme must be enlarged to space-time. Convex closure of the kitchen in a certain space-time region will contain the space-time process of peeling the apples, but now spatial points are replaced by four-dimensional space-time points.

A way of seeing that space-time, not merely space, is required is to ask for the satisfiability or truth conditions of the kinds of sentences just considered. A mere point or region in space is not enough to confirm that an appropriate process took place, such as walking or peeling apples; a space-time region is required. Casually, we often think of evaluating the satisfaction of such commands just in terms of results, but obviously for full confirmation of the appropriate process taking place, examination only of end results is not enough.

Some sentences make the process aspect even more prominent, as in the following examples.

*Go to the store, but walk in the shade.*

*Run under the bridge.*

*Walk back and forth in front of the entrance.*

*Watch the ball roll on the table.*

The relatively unsatisfactory state of the current semantic analysis of process verbs is perhaps partly due to their involving a rich framework of concepts, of which we have sketched only the geometrical space-time part. The semantical analysis of these examples is rather complex. For this reason, various levels of abstraction are needed. In planning an action we often ignore the temporal aspects and consider only the spatial features. For example, in going to the store we may think only of the route in terms of a spatial map. The more detailed examples of actions of getting and putting considered later make such an abstraction.

*Topology.* We have reserved for last our discussion of spatial prepositions invariant under any homeomorphism, i.e., any continuous transformation, because of the much greater generality of topological concepts. All of the geometries considered up to this point assume at least affine space, which
in many ways is part of the implicit background of most ordinary talk. Moreover, in the experience of a young child learning language, and its way around the world as well, topological, affine, metric, Euclidean, and projective notions are not nicely separated, but intermingled from the beginning.

All the same, as part of the geometrical analysis that is the focus of this article, it is germane to ask which common spatial prepositions are, in at least some of their uses, purely topological in character. The obvious example is the use of in to express enclosure in a two- or three-dimensional region. *The horse is in the south pasture* would ordinarily mean the horse was contained by a continuous fence in a certain two-dimensional region. *The button is in the box with a white lid* would ordinarily mean the button was enclosed by the continuous surface of the box. The same purely topological analysis does not hold for an object inside a box without a lid, the case discussed earlier. Notice that the ordinary language user surely shifts between these two kinds of enclosure without consciously noting it.

A less common example, but nicely illustrative of a topological concept, is found in the use of the compound preposition over and under. In tying a knot, one piece of rope must go over and under the other. Similar examples work for knitting and weaving. The familiar complex prepositional phrase in contact with expresses the topological concept of contiguity, and its negation, separation.

*The metal is in contact with the wood at this point.*

*The difficulty is that the first gear is not in contact with the second.*

A more prominent place for topology in linguistic analysis has been argued for by Thom (1970, 1973a, 1973b). In (1973a) especially he argues for a geometrical interpretation of language generally, and particularly of the traditional grammatical categories of noun, verb, and adjective, but these considerations go beyond the scope of this article. The idea most relevant here is set forth in the 1970 article. Thom proposes that the topological classification of structures, especially their geometrical points of singularity, be used to interpret the semantics of sentences describing various actions or events, e.g., *John throws a stone*, but in its present tentative development the classification does not seem to bear at all directly on the semantics of spatial prepositions.

*Problem of context.* No doubt exceptions to our proposed geometrical definitions of various spatial locatives have already occurred to the reader. The reason, we think, is obvious. Except for the most straightforward cases, the meaning of a spatial locative is only finally determined by the context of use. Consider again the case of in, for which we have already noted some difficulties of applying in an unrestricted manner our definition in terms of convex closure. (i) *She is in Italy.* How do we think about the convex closure of a country? (ii) *I saw joy in her countenance.* Even though visual appearance, as a way of expressing emotion, is the issue, we do not seem to have in the phrase *in her countenance* a literal geometrical use of in. (iii) *The infec-
tion is in his arm. Here the geometrical sense of containment is kept, but a phenomenon rather than an object is contained in the arm, and so the definition of being a subset of the convex closure of the arm does not work. (iv) Your daughter is sitting in the dirt. In this case a geometrical relation is denoted, but not in terms of the basic meaning of in. (v) The prisoner is in chains. Again a geometrical relation is denoted but one more complicated than containment in the convex closure of an object.

The general sense of in is found in all these examples, but a correct and accurate semantic analysis must depend on many particular contextual features. Furthermore, these contextual features include the immediately adjacent words, the prior discourse, the perceptual scene in front of the speaker, writer, listener, or reader, etc. What applies to in applies to the other spatial locatives as well, as far as context is concerned. We touch briefly on some of these problems of context in the detailed examples that follow, and we return to the topic in our concluding remarks.

We conclude this section with a table showing the geometrical distinctions we have drawn and examples of prepositional use that fall in each category. Several of the examples will be discussed in detail in the next section.

**TABLE 1**

*Kinds of geometry and examples of prepositional use.*

<table>
<thead>
<tr>
<th>Kinds of geometry</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topology</td>
<td>The pencil is in the box. (box closed)</td>
</tr>
<tr>
<td></td>
<td>One piece of rope goes over and under the other.</td>
</tr>
<tr>
<td>Affine geometry</td>
<td>The pencil is in the box. (box open)</td>
</tr>
<tr>
<td></td>
<td>Mary is sitting between José and María.</td>
</tr>
<tr>
<td>Euclidean geometry</td>
<td>The pencil is near the box.</td>
</tr>
<tr>
<td>The geometry of oriented physical space</td>
<td>The book is on the table.</td>
</tr>
<tr>
<td></td>
<td>Adjust the lamp over the table.</td>
</tr>
<tr>
<td>Projective geometry</td>
<td>The post office is over the hill.</td>
</tr>
<tr>
<td></td>
<td>The cup is to the left of the plate.</td>
</tr>
<tr>
<td>Geometries that include figures and shapes with orienting axes</td>
<td>The dog is in front of the house.</td>
</tr>
<tr>
<td>Geometry of classical space-time</td>
<td>She peeled apples in the kitchen.</td>
</tr>
</tbody>
</table>

**SOME DETAILED EXAMPLES**

In this section we discuss two pairs of examples. The first pair uses the locative near, which was discussed earlier. Our aim is to delve a little deeper into
the semantics of near and to place our inquiry within a specific semantic enterprise, that of instructable robots. As we have done in other articles, we ask what would be required for a robot to interpret commands addressed in a natural language such as English. The second pair of examples uses the locative over. Here our aim is to show how an interesting problem posed by Cresswell can be solved within the geometrical framework we propose. For both pairs of examples, we also take a look at some aspects of the interaction between syntax and semantics.

These examples bring out an important geometrical feature of spatial prepositions. There is, as in other aspects of geometry, a contrast between a general property, e.g., being the bisector of a given line segment, and the particular procedure or process by which the property is generated or verified. This distinction, which, within the history of geometry, comes to the distinction between theorems and constructions, was already important and the subject of much discussion in ancient Greek geometry. Here the distinction is evident in the contrast between our earlier general analysis of the preposition near and the two procedures for near given below which arise from the contrasting actions of getting and putting.

Much of our preceding discussion has contained expressions that describe static locations—The red book is next to the blue book, Find the pencil in the drawer, He chopped onions at the sink. In contrast, prepositions are often used in expressions that describe a change of location—The dog went under the table, Put the red book on the blue book, Go around the table. This distinction is a familiar one in studies of the prepositions in English but the change-of-location cases tend to be neglected. We will examine both kinds of prepositional usage in our first pair of examples.

The robot we feature in our discussion consists of a manipulator (somewhat approximating the human arm in its shape and form) equipped with a simple gripper. For our remarks on spatial expressions, the only details of robot control and operation we need to introduce are the following. (For further details and a more extended discussion of the specific problems involved in natural-language communication with a robot, see Suppes and Crangle [1988] and Crangle [1989].) The robot system has several primitive actions: open-gripper, close-gripper, move-gripper-to-region, gripper-back, gripper-up, move-gripper-direction-distance, and so on. From these primitive actions we build up higher-level basic actions. Three examples are PUT, GET, and LOCATE. We describe each briefly.

The PUT action takes two arguments: item and region. As suggested by these labels, the first argument specifies what is to be placed by the manipulator and the second specifies where it is to be placed. The region must be a horizontal surface. Note that we have a target region, not point, because natural-language spatial expressions very seldom refer to specific points in space. Consider the expressions on the floor, under the canopy, near the door, for instance. These expressions do not on their
own specify any one point but rather a region within which any one point could be selected.

The GET action takes one argument, *item*, which specifies the item that must be located and grasped.

The LOCATE action takes one argument, *item*, which specifies the item whose position must be determined.

A few further remarks are needed on the PUT and GET actions. The PUT action checks that the item specified by its first argument is already in the gripper. If it is not, the GET action is performed. The gripper is then moved to a point immediately above the nearest point of the target region. The gripper is then opened and moved back, thus allowing the item in its grasp to come to rest on the surface.

The GET action uses the LOCATE action to locate the item specified by its argument, moves the gripper to that item, opens the gripper, positions it in the appropriate grasp configuration for that object (a book, a cup, and a fork will each need to be grasped differently, for instance), closes the gripper and then moves the gripper up some small distance with the item firmly in its grasp.

Now consider the following pair of English commands addressed to this robot:

*Put the knife near the plate.*

*Get the knife near the plate.*

In the first command, the expression *near the plate* describes the target region for the robot's placement of the knife. In the second command, the expression *near the plate* describes a spatial restriction that helps the robot select the knife it must get. In rough terms, the robot's interpretation of the commands is PUT(the-knife, near-the-plate) and GET(the-knife-that-is-near-the-plate).

We have of course chosen the basic robot actions PUT and GET precisely for the verbs *put* and *get* of our examples. These verbs are not analyzed further in our discussion but we emphasize that it is possible and indeed necessary to do so in general. The reason we adopt this level of analysis is that our interest here is in the locative *near* and the semantic constraints it is subject to in the context of the actions of putting and getting.

We will use square braces to show denotations; [get] therefore indicates the denotation of the word get. We write GET(*item*) and PUT(*item*, *region*) for the two robot actions that are the denotations of the verbs *get* and *put*. Further, we have [the plate] = S, the set of all plates in the physical environment, and [the knife] = K, the set of all knives in the physical environment.

The question of interest now is what is the denotation of *near*? We answer the question by asking what it needs to be for the interpretation of the get-command and the put-command. For the get-command, the denotation of *the knife near the plate* must be some $k \in K$ such that for some particular $s \in$
S the minimum distance between \( k \) and \( s \) is less than some \( \epsilon \). (This description follows from our earlier discussion of \textit{near}.) We define a function \( g_{\text{near}} \) of two arguments to select this element. Intuitively, the function ensures that some minimal distance \( \epsilon \) exists between pairs of plates and knives if the one item in the pair is said to be near the other. When the phrase refers unambiguously in the given physical environment at the time the phrase is used, only one pair satisfies the requirement. It is the knife in that pair that is selected as the denotation of \textit{the knife near the plate}. The denotation of \textit{the knife near the plate} will thus be \( g_{\text{near}}([\text{the knife}], [\text{the plate}]) \) where

\[
g_{\text{near}}(K,S) = \{k : O < D(s,k) < \epsilon \text{ and } s \in S \text{ and } k \in K\}.
\]

We define \( D \) in terms of the metric \( d \) given in our earlier discussion of the word \textit{near}. That is, the distance \( D \) between two bodies \( s \) and \( k \) is the minimum distance \( d \) between the points \( s \), of \( s \) and \( k \) of \( k \).

\[
D(s,k) = \min_{x \in S, y \in K} d(s,k).
\]

(Keep in mind that \( S \) is the set of plates, \( K \) the set of knives, and \( s \) and \( k \) are individual plates and knives.) As before, we assume that \( \epsilon \) is at the least a function of the size of the two bodies \( s \) and \( k \). Note that the phrase \textit{the knife near the plate} is most likely to refer unambiguously if only one plate had already been selected as the denotation of \textit{the plate}. Often, this condition does indeed hold; either there is only one plate in the environment or a particular plate is already established as the focus of attention at the time the phrase is used. It is important to note too, however, that we do not want to define \( g_{\text{near}} \) in terms of some one particular reference object (i.e., some one plate in this case) under the assumption that there is always a unique reference object. There might be several plates and several knives but only one knife that is near a plate. The function \( g_{\text{near}} \) must be defined over sets.

Turning now to the \textit{put}-command, the denotation of \textit{near the plate} must be some region defined with respect to the geometry of the plate. We define a function \( f_{\text{near}} \) for this purpose. Then \([\text{near the plate}] = f_{\text{near}}([\text{the plate}]) \) where the value of the function \( f_{\text{near}} \) is the region bordering the plate uniformly in the plane of the flat surface holding the plate, with the border being in width no more than the radius of the plate. Note that the size of the region is directly related to the size of the plate. Were we to take into account plates other than round ones—square and oval plates are not unusual—we would redescribe the computation to take shape into account. The function \( f_{\text{near}} \) takes a set as its argument. This set contains just one element when the prepositional noun phrase refers unambiguously at the time it is used.

We now give tree diagrams for the interpretation of our two commands. An interpretation tree is generated from a grammar that has been extended by appending to each production rule at most one semantic function. This function stipulates how a node in the tree obtains its denotation from the denotations of its daughter nodes. In our trees, the category symbols at the nodes are VP for verb phrase, V for verb, NP for noun phrase, N for noun, DA for
definite article, Nom for nominal, and Adv for adverbial. Prep_D stands for preposition of distance, the preposition appearing in the get-command, and Prep_R for preposition of region, the preposition in the put-command. To the right of the colon at each node in the tree we show the denotation at the node.

The grammar fragment needed to generate the two interpretation trees is as follows. We use the subscripted capitals U_1, U_2 for sets of objects (all objects in a set are of the same kind), and the mnemonics it and reg for single-element sets of objects and single-element sets of two-dimensional regions respectively. Note that the semantic functions associated with the two verb-phrase rules and the two preposition rules use the operation of function application.

<table>
<thead>
<tr>
<th>Production Rule</th>
<th>Semantic Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>VP → V Nom Adv</td>
<td>[VP] = [V] ([Nom],[Adv])</td>
</tr>
<tr>
<td>VP → V NP</td>
<td>[VP] = [V] ([NP])</td>
</tr>
<tr>
<td>Nom → DA N</td>
<td>[NOM] = [N]</td>
</tr>
<tr>
<td>NP → Nom_1 Prep_D Nom_2</td>
<td>[NP] = [Prep_D] ([Nom_1],[Nom_2])</td>
</tr>
<tr>
<td>V → get</td>
<td>[V] = GET(it)</td>
</tr>
<tr>
<td>V → put</td>
<td>[V] = PUT (it, reg)</td>
</tr>
<tr>
<td>Prep_D → near</td>
<td>[Prep_D] = g_{near} (U_1,U_2)</td>
</tr>
<tr>
<td>Prep_R → near</td>
<td>[Prep_R] = f_{near} (U_1)</td>
</tr>
<tr>
<td>N → knife</td>
<td>[N] = K = the set of knives</td>
</tr>
<tr>
<td>N → plate</td>
<td>[N] = S = the set of plates</td>
</tr>
<tr>
<td>DA → the</td>
<td></td>
</tr>
</tbody>
</table>

Get the knife near the plate

\[
\text{VP: } \text{GET}([\text{NP}]) = \text{GET}(g_{\text{near}}(K,S))
\]

\[
\text{V: } \text{GET}(\text{it})
\]

\[
\text{NP: } g_{\text{near}}([\text{Nom}_1],[\text{Nom}_2]) = g_{\text{near}}(K,S)
\]

\[
\text{Nom}_1: [N] = K
\]

\[
\text{Nom}_2: [N] = S
\]

\[
\text{DA: } \text{the}
\]

\[
\text{N: } \text{knife}
\]

\[
\text{near}
\]

\[
\text{Prep}_D: g_{\text{near}}(U_1,U_2)
\]

\[
\text{DA: } \text{the}
\]

\[
\text{N: } \text{plate}
\]
Put the knife near the plate

We make one final set of remarks on the preposition near in the command Put the knife near the plate. Although we have shown the denotation of near to be a function of one argument, [the plate], there is good reason to think that it should in fact be a function of both [the plate] and [the knife], for we ordinarily think that the size, shape, and, particularly, orienting axes of the object to be placed have something to do with correct placement near the reference object. A geometry that includes figures and shapes with orienting axes would therefore be required, not merely Euclidean geometry.

We now consider another pair of English expressions. This time they are both simple noun phrases but ones that pose an interesting problem. Our concern in this discussion is to make explicit just a few of the many contextual factors that come into play to help specify the appropriate interpretation of the English. We emphasize that the contextual details come into play at the lexical level, that is, at the level of the individual locatives. We emphasize this point because appeals to context are not new within the philosophy of language but the more usual assumption is that contextual factors operate at the sentence level. Our assumption is that they intrude at the lexical level. The example we use here is the word over. What is particularly interesting about this word is that an adequate account of its use, even in the restricted cases we
consider, requires a geometrical framework like the one we have given in this article.

At first glance, several different uses of over seem hard to reconcile. In the lamp over the table we see a lamp hanging vertically above some portion of the table. And in the axe over his head we see an axe held in place vertically above someone’s neck. However, in the post office over the hill we see the post office on the other side of the hill relative to some reference point roughly in the same horizontal plane. The simple notion of vertically above is not at work here but, at least in part, the notion of horizontally across and to the other side of. In geometrical terms, the basic idea is that of a point of view (or station point) in the horizontal plane and a body whose far side relative to the station point defines a point along the line of projection.

It is useful at this stage to introduce Cresswell’s discussion of the word over because he ends his discussion with this puzzle—how to reconcile the two uses of over—that we believe can be solved. We emphasize that it is worth reconciling the two uses not because we expect there to be some one overarching notion of over that should prevail but because there are cases, and our pair of noun phrases presents such a case, where the surface structure of the over-expression does not indicate which use of over is intended. For our interpretation trees, therefore, we require there to be the one denotation of over that suffices for both uses.

Cresswell suggests that for many prepositions (he discusses across, through, along, around, beyond, past, behind, and over) the basic semantical idea is that of a hypothetical journey which an observer would have to make to be ‘where the action is’ as he puts it. With over, however, Cresswell feels forced to distinguish between a journey and a non-journey sense of the word, a prime example for him of a non-journey sense being the lamp over the table. But in fact it is possible to reconcile the two uses of the word if the primacy of verticality is acknowledged. In terms of Cresswell’s journeys, the journey involved in the interpretation of the lamp over the table would start at the table, proceed in the direction vertically up, and finish at the lamp. The journey involved in the interpretation of the post office over the hill would start on one side of the hill and move vertically up then horizontally across to the other side of the hill. It is important not to neglect the vertical component of this journey because the journey is not through the hill but up to some higher point and then to the other side. The reference object in these cases typically forms some barrier which must be traversed without being penetrated.

In terms of our geometrical distinctions, these remarks point out that over must be interpreted within oriented physical space, not simple Euclidean geometry, and in fact that principles of projective geometry may also have to be used. The puzzle that over then presents is the choice between the simple relation vertically above the reference object and the compound relation vertically up from a station point and horizontally across to the other side of the reference object, which may be expressed as the choice between the geometry of oriented physical space and projective geometry. We will use $h_1$ for the
first, simple relation and \( h_2 \) for the second. \( h_1 \) will in fact be a function of two arguments, the reference object and the located object, and \( h_2 \) will be a function of three arguments, the reference object, the located object, and the station point.

The choice between \( h_1 \) and \( h_2 \) can be resolved by taking into account more of the context than Cresswell did. Consider the idea, now standard within many studies of the locatives, that for expressions of the form \( \text{the } x <\text{locative}> \text{ the } y \), it is generally the case that \( y \) is large relative to \( x \) and less mobile than \( x \). The idea has commonsense appeal in that if \( y \) is to be used to locate \( x \), its greater size and relative fixity would increase the likelihood of its own location being known beforehand. In the case of \( \text{over} \), when this condition holds, the natural reading of \( x \text{ over } y \) is one in which \( x \) is \textit{vertically up from some station point and horizontally across to the other side of } y, our relation \( h_2 \). If the condition does not hold—that is, \( x \) and \( y \) are of the same general size and exhibit similar degrees of fixity—\( \text{over} \) is more naturally interpreted using the simple relation of \textit{vertically above}, our relation \( h_1 \). Returning to our earlier examples, in \( \text{the axe over his head} \) we have two objects of similar size and degree of fixity, whereas in \( \text{the post office over the hill} \) the reference object is larger than the located object. And in other examples, \( \text{the dog over the road} \), for instance, the reference object is considerably less mobile than the located object.

In our previous pair of examples, the commands to a robot, we saw how factors such as size and shape of the reference object came into play to specify the appropriate interpretation of \( \text{near} \). The function \( f_{\text{near}} \) computed a region the dimensions of which depended on the size and shape of the plate that was specified as its argument. And the function \( g_{\text{near}} \) computed a distance relation that was defined in terms of the size of both the reference and located objects. Here we have similar contextual factors at work to select the appropriate geometrical computation for \( \text{over} \)—whether the simple one of vertically above the reference object or the more complex one of vertically up from a station point and horizontally across to the other side of the reference object. It is important to note that there are no clues in the surface structure of the two expressions as to which geometrical computation is appropriate. Both are simple noun phrases consisting of a preposition flanked on either side by a nominal. Another point to note is that the station point is itself often fixed by the wider context. The appropriate response to the command \( \text{Go to the house over the river} \) will depend on where the speaker and hearer are and what reference location—the speaker’s or the hearer’s or some other point altogether—is established in the conversation. Often the command itself will specify that information, as in \( \text{Go to the house over the river from you} \) or \( \text{Go to the house over the river from Sue’s cabin} \).

In the interpretation trees for \( \text{the lamp over the table} \) and \( \text{the post office over the hill} \), we therefore show the denotation of \( \text{over} \) to be the following function of three arguments: \( H_{\text{over}}(R_1,L_1,v) \) where \( R_1 \) is a structure including the set of objects one of which is intended to function as the reference object,
$L_1$ is a structure whose set of objects has the property that one of them is to be located, and $v$ is a contextually determined point of view.

The value of the function $H_{over}$ is one of the two functions $h_1$ and $h_2$ depending on its arguments $R_1$ and $L_1$. Two comments are needed. First, the third argument, $v$, must still be included because it will, as pointed out above, sometimes be set in the language itself, as in the post office over the hill from you. Second, we need structures not ordinary sets because structures can extensionally represent properties of the objects. The properties we are interested in are those already mentioned, namely, size and relative mobility.

$$H_{over}(R_1,L_1,v) = \begin{cases} 
  h_1(R_1,L_1) & \text{if } R_1\text{-type objects and } L_1\text{-type objects are of the same general size and degree of fixity} \\
  h_2(R_1,L_1,v) & \text{if } R_1\text{-type objects are much larger than } L_1\text{-type objects or much less mobile} 
\end{cases}$$

The denotations of the lamp over the table and the post office over the hill are thus $h_1([\text{the table}], [\text{the lamp}])$ and $h_2([\text{the hill}], [\text{the post office}], v)$ respectively. As with the functions $f_{near}$ and $g_{near}$, $h_1$ and $h_2$ must be defined in terms of sets of objects. These sets will in many instances be single-element sets.

We introduce the category Prepo for the preposition over. We regard post office as a compound noun and give it the category $N$ without further comment. We add to the grammar fragment given earlier the following rules.

\begin{align*}
\text{Production Rule} & \quad \text{Semantic Function} \\
\ NP & \rightarrow \ Nom_1 \ Prepo_0 \ Nom_2 \quad [NP] = [Prepo_0] ([Nom_2],[Nom_1],v) \\
\ Prepo_0 & \rightarrow \ over \quad [Prepo_0] = H_{over}(R_1,L_1,v) \\
\ N & \rightarrow \ lamp \quad [N] = J = \text{the set of lamps} \\
\ N & \rightarrow \ table \quad [N] = T = \text{the set of tables} \\
\ N & \rightarrow \ post\ office \quad [N] = Q = \text{the set of post offices} \\
\ N & \rightarrow \ hill \quad [N] = W = \text{the set of hills}
\end{align*}

Here are the two interpretation trees. At the top of each tree we show the result of applying the $H_{over}$ function to its arguments.
CONCLUDING REMARKS

Although this article is full of detailed comments and suggestions, it is far from offering a worked-out semantic theory of spatial prepositions. Much additional work is needed to complete the geometrical analysis begun. Moreover, here, as in related papers we have written, we are sensitive to the importance of context, the theory of which is only in the early stages of development.

One important aspect of context we have not utilized here, but hope to in the future, is the introduction of naive physics, i.e., the physics of ordinary experience (Hayes 1978, 1985). Physics, compared to geometry, is the study of
context. This ball will not roll into the pocket because it is blocked by the red ball. The pure spatial geometry of the ball is untouched by context. The dynamics of its motion is completely determined by it.

The full meaning of many spatial prepositions requires physics as well as geometry. We have already noted that many uses of on imply a physical notion of support. In John ran into the tree and hurt his head, the meaning of into is closer to the physical concept of impact than the geometrical concept of enclosure, as in Mary ran into the house. But more extended analysis of the necessary physical concepts we must leave for another occasion.

Notes

*The authors acknowledge the support of the Spencer Foundation in the work described in the article.

1. The writing samples we refer to were collected as part of a study sponsored by the Spencer Foundation on the writing of high school students in the San Francisco Bay Area, 1986–88.

References


