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Congruence of Meaning*

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1. *Geometric Congruence*

A large literature in philosophy attempts to give criteria for the identity of two propositions. Those who do not like talk about propositions have been much involved in the closely related problem of stating when two sentences or expressions are synonymous. A good review of the earlier work may be found in Quine's *Word and Object* (1960). The contributions of Church, Mates, Sheffler, and others show how difficult it is to get an appropriate concept of synonymy of expressions or criterion of identity for propositions. The efforts of Carnap, for example, to develop a concept of intensional isomorphism in *Meaning and Necessity* (1947) was not brought to a finished state. To a large extent, the same difficulties arise in giving a criterion of identity for proofs, with about as little progress in the case of proofs as in the case of propositions (Recently, almost the same difficulties have been faced again in trying to say when two computer programs are identical.)

The theme I shall develop today is that by looking at the history of geometry and the concept of congruence in geometry we can get a new perspective on how to think about the closeness in meaning of two sentences. (Hereafter, to avoid any commitment to propositions, I shall talk about sentences and not about propositions.) I shall not try to say when two sentences express the same proposition or when two sentences have the same meaning, but rather shall talk about the congruence of meaning of two sentences or expressions. I say *expressions*, because the concepts I introduce need not be restricted to sentences but can deal with noun phrases, verb phrases, and so forth.

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In the long history of the concept of equality or congruence in geometry, there is almost no discussion of the criterion of identity for two figures. Of course, for many formal treatments of geometry, the concept of identity follows directly from the logic of identity in first-order logic, together with the definitions of concepts like those of triangle, quadrilateral, etc. The important point is that the criterion of identity is not an issue in geometry and is not the important or significant concept.

What does have a long and interesting history in geometry is the concept of congruence. Because the axiomatic treatment of the concept is obscure and unsatisfactory in Greek geometry, I shall not review the role of superposition of figures in Euclid. The Euclidean notion of superposition expressed cryptically in Greek geometry is given an admirable intuitive formulation in Kiselyov's well-known Russian textbook on plane geometry: "Two geometric figures are said to be congruent if one figure, by being moved in space, can be made to coincide with the second figure so that the two figures coincide in all their parts."

The theory of congruence for Euclidean geometry was put on a rigorous and explicit basis at the end of the nineteenth century by Hilbert and others. Intuitively, Hilbert's concept of congruence is such that any two figures with the same shape and size are congruent. The important fact for purposes of later discussion is that any two figures of the same shape and size can be related by what is called in geometry a rigid motion. This means that we can transform the spatial origin and orientation as well as the handedness of the axes of reference, without changing the size or shape of a figure. From the standpoint of ordinary experience, one can certainly see demanding a stronger sense of congruence than that characterized by Hilbert. We could, for instance, require that congruent figures also have the same orientation. So, for example, if a triangle has a horizontal base, then any triangle congruent to it must also have its base oriented along the horizontal. To do this, of course, is to strengthen Euclidean geometry, which has no preferred direction and therefore no nonarbitrary definition of *horizontal*. It is straightforward, however, to introduce such directions in geometry, and we all recognize that the absence of a sense of preferred direction in Euclidean geometry is an abstraction from our ordinary ways of thinking about space.

On the other hand, we can move in the opposite direction and develop weaker concepts of congruence. The next, most natural weaker concept is that of two figures being congruent if they have the same shape, but not necessarily the same size. This concept of congruence is ordinarily termed *similarity of figures*.

From this definition we can move on to the concept of congruence in affine spaces. Roughly speaking, in affine spaces lines are carried into lines and, consequently, triangles into triangles, but the shape and size of the triangles are not preserved, and in a general affine space any two triangles are congruent. This weakening of the concept of geometrical congruence can proceed much further A significant example is topological congruence. Two figures are topologically congruent when one is a homeomorphic image of the other, that is, one can be obtained from the other by a one-one bicontinuous transformation. In this case, for example, a square and a triangle are topologically congruent. On the other hand, dimensionality is preserved under topological congruence, and therefore a sphere is not homeomorphic to a circle or a pyramid to a triangle. Beyond topological congruence we can go on to the broadest concept of congruence, namely, that which is preserved under one-one transformations In this case, cardinality is preserved but not much else. Thus, for example, a line segment is in this one-one sense congruent to a square, etc.

Each of these concepts of congruence in geometry, some weak and some strong, has a useful and important role, both in geometrical theory and in widespread applications of geometry to physics and other sciences. It is not my purpose here to make a case for the significance of the concept of congruence in geometry, for it will be generally accepted without much argument. Rather, my purpose is to work on an analogy and to develop corresponding strong and weak definitions of congruence of meaning for sentences or even expressions that are not sentences

Before looking at some examples that will motivate the definitions I want to give, let me interject that I intend to keep the treatment of these matters reasonably informal and reserve the technical and formal presentation of the concepts for another occasion.

Consider first the pair of sentences:

(1) *The book is red*
Le livre est rouge

In spite of general problems about translating from one language to another, we all recognize the closeness in meaning of these two sentences, and my purpose is to give definitions that catch this closeness.

As a second pair, consider the following:

(2) *John and Mary are here*
Mary and John are here

In the case of this pair, we recognize that commuting the order of the proper names in the noun phrase *John and Mary* makes little difference in the literal meaning of the sentence. The closeness in meaning in this case, however, is different from the closeness of the first pair of sentences.

Consider next the pair of sentences:

(3) *John has three apples.*
John has more than two, but less than four apples

In this case the content of the two sentences is very similar, but the second is more pedantic and elaborate in formulation than the first We can probably agree that the second sentence is an approximate *paraphrase* of the first.

2. *Theoretical Framework*

As in the analysis of congruence in geometry, a definite and concrete set of proposals about congruence of meaning depends essentially on the kind of theoretical framework assumed. For the analysis in this paper, I shall assume a fixed, context-free grammar. Such a grammar consists of a finite vocabulary of which a given subset is the nonterminal vocabulary, a set of production rules that have the restricted form required for a context-free grammar, and a start symbol usually labeled *S* (for *sentence*) These ideas are familiar and have been around for more than a decade (Chomsky, 1956, 1959) What is less familiar is the semantical apparatus that I shall assume. The details of the semantical setup are given in an earlier paper (Suppes, 1971); therefore, I shall not repeat all the detailed definitions, but rather shall give an intuitive sense of the main ideas. (The main predecessors of my approach to the semantics of context-free languages are to be found in the literature on computer programming languages, in particular, Irons (1961) and Knuth (1968). I have also been influenced by the work of Montague (1970, 1973) on English as a formal language.)

The context-free semantics that is added to the context-free grammar, and that is closely wedded to the grammar, consists of two main parts. One part consists of giving a model structure in the sense of classical model theory in order to assign a reference (relative to a model) to various terminal words, although not necessarily to all terminal words. (By *terminal word* I mean the ordinary words of the language and not the nonterminal grammatical vocabulary like *noun phrase, verb phrase, intransitive verb*) The important point is that a model structure consists of a nonempty domain D of individuals and an evaluation function that assigns a denotation to each terminal word. The denotation of a word is a set-theoretical object that is part of the natural Zermelo hierarchy built up from the domain by taking sets of objects, sets of sets of objects, and so forth. I shall not have more to say about the model structure, because I do not want to enter into the technical definitions relevant to the construction. The intuitive idea is straightforward and a natural extension of Tarskian semantics for first-order theories.

A more important and interesting point in the application of model-theoretic semantics to natural languages is that set-theoretical functions must enter in telling us how denotations of the various parts of the sentences are related. The analysis of how the various parts of a sentence are related in terms of meaning, that is, what set-theoretical functions relate the denotations of the words occurring in the sentences, constitutes one important part of our intuitive idea of meaning. Like the denotations of individual words, the set-theoretical functions that relate the denotations of individual words are ordinarily relatively simple in character. If, for example, I use the phrase *red flowers*, then the natural set-theoretical function for this phrase is the intersection of the set of red things and the set of flowers.

The problem is how to bring order into the method for introducing the set-theoretical functions relating the parts. Fortunately, a completely straightforward answer is available for context-free languages. With each production rule of the grammar we associate a semantic function, and thus we may convert each derivation tree of the grammar into a *semantic tree* by attaching not only labels to the nodes of the tree, but also denotations generated by the semantic functions. (The idea of identifying the meaning of a sentence with an appropriate tree is developed rather thoroughly in terms of categorial grammars, but in a different direction from the consideration of congruence of meaning by Lewis, 1970.)

In previous writings I have termed the grammar and model structure *simple* if the following conditions are met: Each terminal word has a denotation, and each production rule of the grammar has exactly one semantic function associated with it. There is no reason to insist that simple grammars have a position of widespread applicability; I mention them only because they give a feeling for the natural place to begin the analysis. It is easy to move on to more complicated characterizations.

All this is by way of preliminary analysis; let me quickly summarize what I have said. First, I introduce model structures that are fairly natural extensions of Tarskian relational structures for first-order theories to serve as the model structures for context-free languages. Second, a semantic function is assigned to each production rule of the grammar. By this means, any derivation tree of the grammar is converted to a semantic tree. In the simple case, a semantic tree is merely a derivation tree with the addition of a denotation for each node.

A brief remark about grammatical and semantical ambiguity is needed. It is often the case that more than one derivation tree in a given grammar is possible for a sentence. When there are at least two such trees we say that the sentence is grammatically ambiguous. If in addition the denotations of the roots of the trees differ for a fixed model structure, then the sentence is also semantically ambiguous (relative to the given model structure).

A second remark about the class of model structures is also needed. In classical model theory of first-order logic, it is natural ordinarily to consider the set of all possible models of a sentence or of a theory or of a language, but in the context of natural language, it is more appropriate to hold certain aspects of the models constant and to vary only some restricted part. For example, we may in our analysis of paraphrase want to assume that arithmetic is constant across all the models considered and, consequently, restrict the set of models to those in which arithmetic has its standard interpretation. In my view it is a mistake always to test the meaning of a sentence of natural language by asking for its logical consequences. It is often more appropriate and informative to narrow the class of models to those in which variously broadly accepted nonlogical theories like arithmetic are satisfied.

Thus, the definitions of congruence are for a fixed set $\mathcal{M}$ of model structures, not in general for the set of all possible model structures of a language. I also allow for the possibility that a sentence may have more than one semantic tree (up to isomorphism) with respect to the given grammar and a fixed model structure.

3. *Four Definitions of Congruence*

I begin with a strong notion of congruence.

Definition 1. *Let* S_1 *and* S_2 *be sentences of the given language, that is, derivable by means of the given grammar of the* *language. Then S_1 is strongly $\mathcal{M}$-congruent to S_2 if and only if the set of semantic trees of S_1 and S_2 can be made identical with respect to each model structure of $\mathcal{M}$, except perhaps for labeling, by identifying isomorphic trees.*

The force of this definition is that the denotations of each node must be identical, and the tree structures themselves must be isomorphic, but both terminal and nonterminal labels of corresponding nodes can differ. Examples of congruent sentences under this definition are the following:

> *All men are mortal.*
> *Every man is mortal.*

This pair exemplifies the fact that shifting from the plural to the singular should not really affect the meaning of universal affirmative sentences, and thus they should be congruent in a strong sense

On the other hand, if we use a noun phrase rather than an adjective for predication, we cannot satisfy strong congruence in going from the plural to the singular:

> *All men are animals*
>
> *Every man is an animal.*

The second sentence has an additional word, the indefinite article, with no corresponding terminal node in the first sentence, and thus the tree structures are not isomorphic.

If we think of our language as containing both elementary parts of French and Russian, as well as of English, then of the following three sentences that essentially express the same idea in the three languages, the English and French sentences are strongly congruent but the Russian is not, because of the absence of a definite article and the copula:

> *The book is red.*
> *Le livre est rouge*
> *Kniga krasnaya.*

I give now a second definition incomparable to the first. By *incomparable* I mean there exist pairs of sentences that are congruent in the sense of the first definition, but not in the sense of the second, and conversely. I call this second sense permutational congruence in meaning and form.

Definition 2. *Let S_1 and S_2 be sentences of a given language as before. Then S_1 is permutationally $\mathcal{M}$-congruent to S_2 in meaning and form if and only if each semantic tree of S_1 can be obtained from a semantic tree of S_2 by a sequence of permu-*

tations of branches of subtrees for every model structure of $\mathcal{M}$. Thus in the sense of this definition the following sentences, mentioned earlier, are permutationally congruent in meaning and form, but are not congruent in the sense of strong congruence.

> *John and Mary are here.*
> *Mary and John are here*

In this pair we have a natural permutation of the order of proper names in the subject, something we would ordinarily consider fairly unimportant in conveying the sense of the sentence We would also ordinarily treat as permutationally congruent in meaning and form a sentential conjunction that results from another conjunction by interchanging the components, and similarly for a disjunction, but not when an implicit temporal order of events is implied by the order of the components. Consider *They got married and had a baby* versus *They had a baby and got married.*

Looking at the first two definitions, we are naturally led to a third definition that is weaker than either of the other two, namely, that of being permutationally congruent in meaning, but not necessarily in form.

Definition 3. *Let S_1 and S_2 be sentences of a given language. Then S_1 is permutationally $\mathcal{M}$-congruent in meaning to S_2 if and only if each semantic tree of S_1 can be obtained from a semantic tree of S_2, except possibly for labeling, by a sequence of permutations of branches of subtrees for every model structure in $\mathcal{M}$.*

Given this definition, we then have permutational congruence of sentences in different languages, because we are again no longer looking at the labeling itself For example, the following three sentences in French, English, and German would be permutationally congruent.

> *John and Mary are here*
> *Marie et Jean sont ici*
> *Marie und Johann sind hier*

Under the natural grammar for arithmetical expressions, the following pair would also be permutationally congruent:

$$2 + 2 = 4$$
$$4 = 2 + 2$$

Let me guard against one kind of misinterpretation of permutational congruence. It might be thought that simply by permuting the branches of the tree we could show mistakenly that there was permutational congruence of the sentences *John loves Mary* and *Mary loves John*. This is of course not the case, because the semantic function that is the root of the two trees is different for these two sentences.

I turn now to the fourth definition. It is easy to show that each of the three definitions of congruence already given implies $\mathcal{M}$-paraphrase, which is the most general and therefore the weakest concept of congruence I shall introduce.

Definition 4. *Two sentences are $\mathcal{M}$-paraphrases of each other if and only if the roots of their trees denote the same object with respect to every model structure in $\mathcal{M}$.*

If $\mathcal{M}$-paraphrase is replaced by logical paraphrase, we get the definition that Montague, for example, liked: Two sentences are L-paraphrases of each other if and only if they denote the same function from possible worlds to truth values. Closely connected with this latter definition is Frege's characterization of sentences being paraphrases of each other if and only if they have the same logical consequences. It seems to me that the condition of logical consequence or logical paraphrase is too strong. In ordinary language we regard the following two sentences as paraphrases of each other, but of course they are not logical paraphrases:

Mary has three apples and John has four.
Mary has three apples and John has one more.

Unless arithmetic is assumed as a part of logic, these two sentences are not paraphrases of each other in the logical sense, although under the intended treatment of $\mathcal{M}$-paraphrase they would be because arithmetic would be held constant across the set $\mathcal{M}$ of model structures, i.e., the elementary laws of arithmetic would be satisfied in every model structure of $\mathcal{M}$.

Another example on the assumption that arithmetic is not part of logic is the following pair of even simpler sentences:

$2<4$
$4>2$

(at least under the treatment I prefer of definitions as noncreative axioms in the object language).

The four definitions given do not in any sense exhaust the possible definitions of congruence of sentences. They are meant to exhibit the possibilities and to show how we may deal in a natural and simple way with sentences that all of us accept as being close to each other in meaning.

4. *Properties of Congruence*

Turning once again to the classical geometrical tradition, we should be able to ask many questions about congruence of meaning if our concepts of congruence are remotely similar to those that have been so useful in geometry. I have divided this topic into three parts. First, I look at the natural analogues of classical geometric theorems about congruence of figures. Second, I examine the relation between congruences and groups of transformations. A new way of talking about transformational grammars arises from this discussion. Finally, I consider some conjectures about the expressiveness of languages when the congruence relation is that of paraphrase.

A familiar concept in Euclidean geometry is that of congruence of polygons. Two polygons are said to be congruent if there is a one-to-one correspondence between their vertices, so that the corresponding segments and the corresponding angles of the two polygons are in every case congruent to each other. We assume of course in this definition that we already have a characterization of the congruence of segments and the congruence of angles. What is interesting about polygons is that the only rigid polygon is a triangle. The meaning of this is that the only polygon whose shape is determined by its sides alone is the triangle. We may ask a similar question of congruence about sentences, with terminal words corresponding to segments. If two sentences are such that their terminal words are congruent, that is, have the same denotations, under the natural left-to-right ordering, then are the sentences strongly congruent? In other words, is the meaning of sentences within strong congruence rigid with respect to terminal words? It is easy to see that in general the answer is negative for sentences that have two or more words. It is trivial to construct examples of context-free languages to show that this is so.

On the other hand, for a wide variety of formal languages, rigidity of congruence with respect to the terminal symbols of expressions is a fundamental property. Essentially, such rigidity is characteristic of the language of all theories with standard formalization, that is, of all theories formulated in first-order logic with identity, which includes such standard examples as the algebraic theory of fields, elementary number theory, and axiomatic set theory.

By looking at languages for theories with standard formalization, it is easy enough to find languages rigid with respect to strong congruence. Also evident for such languages as ordinarily formulated is that the concept of strong congruence reduces to this. The only sentences strongly congruent to a given sentence are the alphabetic variants of the sentence itself.

Further, in first approximation most sentences of ordinary language used in a literal or scientific sense are rigid. The interesting comparisons are probably between languages that have been subsumed under a single grammar as in the case of machine translation. Although simple examples of rigidity have already been given, and fragments of grammars of English, French, and German can be put together to form a rigid grammar in the sense of strong congruence, the very absence of rigidity in the sense of strong congruence is a major contributing factor to the difficulty of machine translation.

In the case of permutational congruence of meaning for both formal languages and natural languages, we have many congruent sentences. Examples have already been given, but here is another: The first-order theory of commutative groups contains numerous expressions that are permutationally congruent, and of course many of these expressions are congruent just because of the symmetry of the logical predicate of identity. Something that we all recognize as fundamentally conventional is properly reflected in the definition of permutational congruence. For example, the axioms for commutative groups, all of which have as a single predicate identity, can be written with the left-hand and right-hand terms reversed; there are only more or less standard conventions as to what to put on the left side and what on the right side.

Examples of permutational congruence of sentences of ordinary language have already been given, and others may be constructed in terms of sentential connectives or in terms of noun phrases or verb phrases. On the other hand, there seems to be no straightforward generalization of the concept of a language being rigid with respect to strong congruence to its being rigid with respect to permutational congruence. The reason is transparent. If the terminal words of two languages are congruent under a permutation, it does not follow at all that the sentences are permutationally congruent. Perhaps the simplest examples may be constructed from any transitive verb. For instance, as already remarked, *John loves Mary* is not permutationally congruent with *Mary loves John*, even though the terminal words can be put into one-to-one correspondence under a permutation.

I reserve for the moment discussion of the concept of congruence corresponding to paraphrase and turn to the relation between transformations and congruence.

5. *Transformations*

The geometrical analogy developed at the beginning of this paper can be pushed further to suggest a relation between congruence of meaning and transformational grammars that does not yet seem to have been explored in the literature on linguistics and the philosophy of language. Let me briefly review the situation in geometry. Given the idea of motion to obtain superposition of figures, it was gradually realized that a motion may be conceived as a geometric transformation of the plane (or space) and that such a geometric transformation is in its most general form any one-one function mapping the plane (or space) onto itself. The particular transformations that correspond to motions admissible in Euclidean geometry are just the transformations that form what has come to be called the group of rigid motions or Euclidean motions.

Originally, transformations were looked upon as a rigorous way of talking about superpositions. At an early date the connection between transformations and symmetries of figures was also recognized. For example, in late Hellenistic times, Pappus discussed earlier work by Apollonius showing that a transformation by central symmetry, or by circular inversion, would carry a line or a circle into a line or circle. Probably the first person to have a definite idea of using a transformation to determine the properties of a general figure from the simpler properties of a special one was Poncelet (1822) who, at the beginning of the nineteenth century, used projective transformations for purposes of simplification.

The connection, however, between transformations and congruence was set forth in the latter part of the nineteenth century by Felix Klein in his famous inaugural dissertation that formulated his Erlanger Program. To each group of transformations there corresponds a congruence relation, and to each congruence relation there corresponds a group of transformations. Klein's program was to study the significant groups of transformations to identify the congruence relation or, put another way, the geometric properties preserved under the group, and correspondingly, given a congruence relation, to determine the group of transformations under which it remains invariant (see Klein, 1893).

It is sometimes said that Klein's lucid and explicit characterization of the relation between groups of transformations and invariant properties or congruence relations is the most important conceptual contribution to geometry since the ancient Greeks. In any case, the subsequent history of geometry has certainly been deeply affected by his viewpoint, and today it is probably more common to think of a given group of transformations and the properties that are held invariant under this group than it is to think about a particular concept of congruence.

In principle, the same program should be feasible for congruence relations of meaning. Although we can ask for the transformations that preserve the congruence relation, there are several conceptual problems that we must first deal with. In fact the way in which we shall deal with these problems that are in a sense preliminary is not yet accepted or fully agreed upon. The problem is this. In the case of geometry it is easy to say what a transformation is. It is a one-one function mapping the entire space onto itself. We thereby have a simple and straightforward mathematical characterization of transformations as objects. The situation would have been quite different if the attempt had been made to define transformations not on points, but on figures, so that transformations take as their arguments not points, but figures. It is probably intuitively easier to use the latter definition. In talking, for example, about one figure being superimposed or moved to coincide with another, it is not natural to think about transforming the entire space. Physically and empirically we certainly do not think in such terms, but rather in terms of local effects only on the two figures in question. Mathematically, however, it is much simpler to talk about transforming the entire space rather than individual figures.

Geometrically speaking, transformational grammars are more or less currently defined as transformations on figures rather than on points, for in the standard approach, it is customary to define transformations in the linguistic sense as mappings of trees into trees. Thus we can start with a context-free grammar and consider the trees generated by this grammar; the transformations then map these trees into other trees. The attempts to give an exact definition of the concept of transformation, as for example that given by Ginsburg and Partee (1969), is awkward from a mathematical standpoint and certainly encourages the search for a definition closer in spirit to that used in geometry.

Unfortunately, we do not have anything like the natural distance function between points to use in defining transformations on tree nodes, terminal words, or sentences of a language. Certainly, a one-one transformation on the finite set of terminal words is not satisfactory and will not permit the deletions and insertions as required, for instance, in the transformation from the active to the passive voice. There is the possibility of defining the transformations on the sentences, but this is in effect almost the same as defining the transformations on the trees, and the technical reason for choosing the trees rather than the sentences is that the sentences themselves are syntactically ambiguous. Thus the move from trees to sentences is not one that will improve the conceptual situation. If the nodes of the trees are not put in the context of the tree itself, they are too unconnected from other objects and therefore do not seem suitable as objects to be transformed.

One natural suggestion is that transformations should operate on the production rules of the grammar. The reason for doing this is that we can require that the transformation of a production rule use the same semantic function as the original production rule. Thus, when I transform the rule that carries a sentence in the active voice into one with a passive voice, I do not actually change the semantic function that establishes a relation between the denotation of the noun phrase that is the subject, the denotation of the transitive verb, and the denotation of the noun phrase that is the object. The idea of defining transformations on the production rules is closely connected to the concept of a syntax-directed translation scheme in computer science, but it would lead too far afield to develop the relevant formal machinery in this paper.

Establishing a close connection between transformations and congruence relations of meaning does not depend upon the particular definition of transformations just mentioned. One can work with the definition already familiar in the literature, that is, having transformations map trees into trees, and still look for the group of transformations that preserve a given congruence relation.

Without entering into technical details, it is easy to state in an informal way what transformations correspond to strong congruence or permutational congruence. In the case of strong congruence, the group of transformations can be characterized in terms of transformations of individual vocabulary words into other words. In the case of terminal words, the mapping must be into terminal words having the same denotation, and in the case of nonterminal words the mapping must be into nonterminal words that yield isomorphic semantic trees. My point is that strong congruence can be characterized in terms of such point transformations, so to speak, once we apply them to the production rules of the grammar, including the lexical rules.

For permutational congruence, the group of transformations can be characterized in terms of appropriate subgroups of the full permutation group, but different permutations may be applied to different production rules of the grammar. Of course, as in the case of strong congruence, a mapping of terminal words into terminal words with the same denotation is also required.

There is a natural question to ask about the language generated by the group of transformations corresponding to a given concept of congruence. It is especially appropriate to ask this question, because the standard results in the literature indicate that the general concept of transformations mapping trees into trees is far too powerful, in the sense that in applying the transformations to a context-free language we may generate any recursively enumerable set over the given finite vocabulary. Salomaa (1971) has shown that any recursively enumerable language may be generated by a transformational grammar over a regular language, which is much more restricted than a context-free base. In view of the simplicity of the register machines or Turing machines that may be used to generate any partially recursive function over a finite alphabet, it is not surprising that results of the simplicity of Salomaa's are obtainable.

On the other hand, the situation is quite different for highly restricted senses of transformation. For example, the group of transformations corresponding to a strong congruence relation over a regular language leads only to a regular language, and the group of transformations corresponding to strong congruence over a context-free language leads only to a context-free language The transformations corresponding to permutational congruence can lead from a regular language to a context-free language, but not to something more powerful.

It is not my purpose here to present results of this kind in formal detail and to prove appropriate theorems. Thus I have only sketched some of the ways in which the concept of transformation assumes a more restricted character when it is tied to semantical notions, particularly to a semantical congruence relation

The results of Salomaa suggest an interesting conjecture regarding paraphrase. Given the extent to which a regular language can be transformed to generate any recursively enumerable set, and therefore any language over a finite alphabet, it might seem that the expressive power of any language can be paraphrased in the simple structure of a regular language. The syntactic results suggest this as a serious possibility. But the well-known results about the limited power of finite-state machines that correspond to regular languages suggest that once we tie the semantics explicitly to the power of the language as well, no such reduction by paraphrase to a regular language will be possible. We know, for example, that a finite-state machine cannot multiply any two arbitrary integers. Already, this suggests that once we include the semantics of the simple recursive language of arithmetic, we shall not be able to reduce by paraphrase to a regular language. The intuitive argument seems clear, but the formal analysis is not yet completely explicit. Almost certainly the group of transformations corresponding to the very general sense of congruence expressed by the concept of paraphrase will require considerably more effort to characterize than do the groups corresponding to the stronger senses of congruence I have discussed.

6. *Concluding Remarks*

I have tried to outline the beginnings of what I think might properly be called a geometric theory of meaning. It has been remarked by many people that semantical theory as applied to natural language has not yet led to a series of results comparable in depth to those obtained in the theory of models for formal languages. One possible feeling is that this can hardly be expected, because natural languages are fundamentally empirical phenomena in contrast to formal languages, which may be studied as a part of pure mathematics. However, this seems to me a mistake. My hope is that semantical theory or, more generally, the tools of logic, may play the role in the study of natural languages that classical mathematical analysis has played in physics.

My final point is that the emphasis in the philosophy of language should be on analysis and not on reduction. The reduction of much systematic discourse to first-order logic has been important and represents a long tradition that begins with Aristotle. What is more important for the philosophy of language of the future is to concentrate on the analysis of natural language as it is used in practice and not to be concerned with the reduction of that practice to an artificial regime. One direction to move in obtaining greater empirical fidelity is to widen the concept of sentence or sentence utterance to that of speech act. Unfortunately, the theory of speech acts is still in a nascent state. Unlike the theory of language I have been able to draw on in characterizing congruence of meaning, correspondingly clear and definite concepts have not yet been developed for speech acts. Intuitively, significant concepts of congruence are used continually in abstracting sentences from speech acts, but the theory of that abstraction is left wholly informal. Development of an explicit theory of congruence for speech acts is a task for the future, but one that seems far from hopeless. The tools of analysis I have described should be useful in that enterprise as well.

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