LOGIC AS A DIALOGICAL GAME

(An experiment in teaching constructivist logic to

elementary-school and high-school students)

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by

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LOGIC AS A DIALOGICAL GAME*

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Introduction

Professor Paul Lorenzen of Erlangen University, Germany, visited at Stanford University from March 17, 1965 through May 14, 1965. The first section of the report consists of Professor Lorenzen's characterization of his system of operative, constructive logic as a dialogical game. In the second section, Frederick H. Binford describes the pedagogical program conducted at Stanford in connection with Professor Lorenzen's visit.

I. Rules and Playing Procedure

The traditional textbook of logic begins with syllogical rules, as e.g.,

All A's are B's
<u>All B's are C's</u>
All A's are C's

Today the categorized sentences which occur here are analyzed as being composed of prime sentences (of the form Ax), with the help of sentential connectives <u>and</u> quantifiers. So, modern textbooks start with sentential logic. For classical sentential logic there is the well-known truth-table method for deciding upon the logical truth of any composed sentence. In contrast to the constructive (intuitionistic) logic,

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classical sentential logic always presupposes the principle of excluded middle, "A or not A", for all prime sentences. So it would not be necessary to introduce "rules of inference" for classical sentential logic. Nevertheless, such rules are normally used in textbooks for beginners. The choice of a set of such rules is arbitrary, even if one calls the chosen rules "natural". But, one may argue, this is only pedagogically relevant; theoretically, all valid rules are on a par, and the validity of a rule can also be decided upon by truth-tables.

The situation changes if the quantifiers are taken into account. For quantificational logic no decision method exists. So one must use "natural" rules of inference. There is no longer a method of proving the chosen rules valid--one just has to accept the "natural" ones as valid.

The game-theoretic interpretation of logic has been worked out in order to overcome these shortcomings of the modern approach. It gives a justification both of constructive and of classical logic. The basic idea is to define the sentential connectives not by truth-tables, but by their use in dialogues. For the binary composed sentences $S \& T$, $S \vee T$, and $S \rightarrow T$ "attacks" and "responses" are defined by an "attack-response table" as follows.

Composed Sentence	Attack	Response
$S \& T$	L?	S
	R?	T
$S \vee T$	?	S
		or T
$S \rightarrow T$	? S	T
		or attack on S

Each attack consists of an attacking mark (L?, R?, or ?) and, in the case of $S \rightarrow T$, the attacker asserts the antecedent (here, the sentence S).

If one requires of a "primitive" binary sentential connective that in the attack-response table each of the formulas S and T shall occur

C always attacks the sentence of D in the same line. If there is no attacking mark the move is always a response to the last attack of the other player.

As has been proved in the literature (see P. Lorenzen, Metamathematik, 1962), a conclusion can be defended in this game against any strategy of C if and only if the conclusion is constructively implied by the premises (e.g., derivable in the intuitionistic calculus of Heyting). If one adds all premises of the form $S \vee \neg S$, one gets all classically implied conclusions of sentential logic. Similarly, in quantifier logic, if one adds all formulas in the form $\forall x \forall y \ldots (Sxy \ldots \vee \neg Sxy \ldots)$ one gets all classically implied conclusions. Without these premises the constructive logic blocks those conclusions depending on excluded middle and on deriving existence from the denial of a universal.

II. Pedagogical Program

A. Acquainting the staff with constructive logic.

Several of the staff joined in four sessions in which Professor Lorenzen presented his constructivist logic as a dialogical game between two players: a challenger or giver of premises and the defender of a conclusion. In additional smaller sessions, staff members played the game among themselves. Professor Lorenzen also gave special intensive training to Mr. Binford of the staff.

B. Preparation of materials.

On the basis of experience in presenting the game to the Institute staff, to teachers at Peninsula School, to parents of Peninsula School children, to the children themselves, and in frequent conference with Mr. Binford, Professor Lorenzen several times rewrote the statement of the rules for playing the game. Further revisions are anticipated.

Several sets of cards for playing the game were prepared so that each pair of students in the classes could engage in the game.

Several problem sets were prepared for the students to use.

C. Description of Rules.

The students were given the following summary of rules for playing the card game employing the logic described above.

1. <u>The pieces</u>.

   a. As prime sentences, cards bearing one of A, B, C, D, ... (several of each) (or cards bearing simple sentences).

   b. As connectives, cards bearing one of: and, or, not, if, then. (several of each) (or cards with the symbols: &, v, $\neg$, $\rightarrow$)

   c. Attacker: A pointer, chessman, coin or other object with which a player can indicate the point of his attack--one attacker for each player.

2. <u>Sentences</u> (or formulas):

   a. Prime sentences.

   b. Sentences composed as follows: If 'S' and 'T' are sentences, then each of the following is a composed sentence: S and T, S or T, not S, if S then T. (Or with symbols the following are composed formulas: S & T, S v T, $\neg$S, S $\rightarrow$ T.)

3. <u>Format</u>.

There are two players, the Defender and the Challenger. At the beginning of play the Challenger puts some number (usually a small number) of sentences, called premises, on the right side of the board. The Defender puts one sentence, called the conclusion, on the left side.

4. <u>Attacks and responses</u>:

Moves consist of attacks and responses as follows: The Challenger attacks a prime formula of the Defender by putting his attacker on that prime formula. There is no response. The attack may end play. (See 6). The Defender is not allowed to attack a prime formula of the Challenger. Attacks and responses to attacks upon composed formulas are as follows:

Composed Formula	Formula under attack by other player's attacker,*	Response: Attacked player puts on his side of the board:
S and T S & T	S *and T S *& T	S
	S and* T S &* T	T
S or T S v T	S $\overset{*}{\text{or}}$ T S $\overset{*}{\vee}$ T	S or T
not S $\neg$S	$\overset{*}{\text{not}}$ S $\overset{*}{\neg}$S and attacking player must also put 'S' on his side of the board.	Counter attack against S
if S then T S $\rightarrow$ T	if S $\overset{*}{\text{then}}$ T S $\overset{*}{\rightarrow}$ T and attacking player must also put 'S' on his side of the board.	T or counter attack against S

5. The Play:

   a. The Challenger's moves:

      (1) He opens play by attacking the Defender's conclusion.

      (2) He must respond if the Defender has just attacked.

      (3) Otherwise he must attack whatever formula the Defender has just put.

   b. The Defender may as he wishes either:

      (1) Attack any one formula put by the Challenger.

      (2) Respond to the last attack by the Challenger.

   c. The Defender is allowed to repeat an attack he has made before only if:

      (1) The Challenger, since his response to the earlier attack, has put a formula he had not put before, or if

(2) The Defender is himself under an attack different from the one he was under when he first made his attack.

6. Ending Play:

The Defender wins whenever the Challenger has put and is attacking the same prime formula.

The Challenger wins whenever the Defender cannot move when it is his turn.

D. Elementary-school pilot group, Peninsula School

Because of the time limits of Professor Lorenzen's stay it was necessary to locate a pilot group quickly when we were prepared to begin with children. Several years ago Professor Suppes carried on experimental teaching of logic at Peninsula School, a private school with a short chain of command and flexible scheduling; we asked this school to provide us with an experimental group of children. Within a week a class was set up.

During this week, Professor Lorenzen gave an introductory lecture on the operative logic and the dialogical game to the teachers of the school and to the parents.

The class consisted of 12 students from fifth grade through eighth grade. It met for eight sessions of an hour each. Mr. Binford did the teaching with frequent assistance and direction from Professor Lorenzen. However, very little formal teaching is needed once the fairly simple format is grasped, so during most of the time the students played the game under supervision.

In fact this approach to logic requires very few rules; strategy becomes the focus of attention. The effects of various strategies are best learned in actual playing experience.

The class sessions were as follows:

1. Discussion of sentences of different form, where logical form determines how a sentence may be appropriately attacked or defended.

2. Playing the game in English using cards bearing simple English sentences and the connectives 'and' and 'or'.

3. Distribution of copies of two-page typed statement of rules for the game. The entire set of rules was gone over. This was apparently quite too much material for one session. The students seemed overwhelmed, confused, and discouraged.

4,5. The students paired off to play the game with the prepared cards, using a sheet of dittoed problems.

6,7. Return to the rule for conditionals which had caused difficulty in the third class meeting. Continuation of play.

8. An introduction to quantifier logic.

The choice of school in which to conduct the pilot class proved to have been a poor one. An extensive field trip together with the spring recess resulted in a two-week interruption between the third and fourth sessions. Further, attendance was on a voluntary basis at the school, and half of the students did not return for the fourth session. This was certainly partly due to the pedagogical error of presenting too much material in the third session. After the eighth session, the class was terminated.

E. High-school pilot group.

With two weeks remaining in Professor Lorenzen's visit, another more stable pilot group was needed that could be set up very quickly and could progress rapidly. A group of 30 high-school sophomores was organized at Cubberly High School. These students had had a three-weeks' course in sentential truth-functional symbolic logic at the beginning of their geometry course in the autumn. The new class met daily from 12:35 to 1:10 for four days the first week, and for five days the second week. Mr. Binford taught the classes with frequent assistance from Professor Lorenzen. Material was presented in the same order as at the Peninsula School. In addition to the game, extensions of the game for proving validity and invalidity were studied.

After the end of Professor Lorenzen's stay, the students were invited to continue another two weeks with Mr. Binford. Five accepted and continued. For the high-school group, the system was extended in the second two weeks to include quantifier logic. The rules were extended to include attacks and responses to attacks on universal and existential quantifiers. The use of the cards was abandoned in favor of pencil and paper. Informal proofs were studied. Simple theorems of number theory dealing with addition and subtraction were proved. The cancellation law was proved from two different axiom systems and a theorem of formal geometry using four-place and six-place predicates was proved. Geometric interpretation in terms of congruence of segments, angles, and triangles was given only after the theorem was proved.

Our experience with the two pilot groups sustains the hypothesis that this is an efficient, readily teachable approach to logic. The structure of the rules is amazingly simple. In our previous logic project, we used 16 rules of derivation for sentential logic in our elementary-school textbook; only six rules of attack and response are needed for the game. An academic year of three one-half hour lessons a week was required to take fifth graders through a program in classical sentential logic; it is a fair guess that this method would take no more than two or three months. A second year was required simply for the logic of the universal quantifier; again it is a reasonable guess that both universal and existential quantification could be adequately presented in three months.

This suitability for teaching may have misled us into too rapid a presentation at Peninsula School. However, since this approach makes a lesser demand on curriculum time for a complete presentation, it probably enjoys a greater chance for adoption into school curricula. Its simpler rule structure is suitable for study by the average child as well as by gifted children. Finally, its grounding in the everyday meaning of connectives means that it will probably be remembered longer; thus it may influence everyday usage, by encouraging critical attitudes toward discourse and by increasing precision in the use of language.

Lectures to Institute Staff

Professor Lorenzen gave four lecture-seminars to the staff of the Institute for Mathematical Studies in the Social Sciences. In his lectures, he presented his ideas on the foundations of number theory, geometry, logic, and the use of logic in mathematics. The content of these lectures can be found in the references that follow.

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