CONCEPT LEARNING WITH NON-VERBAL GEOMETRICAL STIMULI

by

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Concept Learning with Non-verbal Geometrical Stimuli[1]

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This experiment was a preliminary study of the learning of non-verbal relational concepts by children. The stimuli were simple closed plane figures and the concepts were the geometrical transformations, rotation, reflection and size change or stretch.

Although the literature appears to hold few papers directly relevant to this study, there are several well-documented areas of research that are germane to learning the concept of geometrical transformation. Shape perception and shape discrimination learning are both preliminary to learning concepts where figures are the stimuli. Sutherland (1962)has reviewed much of the shape discrimination literature on animals, but no equivalent review exists on human shape discrimination. The discrimination learning literature on humans is extensive and covers a large variety of experimental manipulation. Two types of discrimination learning studies are closely related to relational concept learning experiments. In size and brightness discrimination problems a relation between stimuli may provide cues for discrimination. In these studies the experimental operations are directed at forming an S-R connection between a particular physical stimulus and some overt response. Transposition studies have shown (Lawrence and De Rivera, 1954) that Ss often respond to the relation (bigger, brighter, etc.) rather than to a physical stimulus of a given magnitude. Similarity and oddity discrimination problems also have the property of the solution hinging on a relation between two or more stimuli (Scott, 1964).

Both of the above types of studies become relational concept experiments if the discriminanda are changed on every trial while the relation between or among the discriminanda is kept constant over trials. A recent study by White (1965) seems to bridge the gap between concept and discrimination learning experiments by utilizing different stimuli on every trial but making small physical changes in the successive stimuli rather than keeping some relationship between the positive and negative stimuli constant over trials.

Two recent studies used sample forms and transformations of the sample forms as stimuli. A study by Gibson, Gibson, Pick and Osser (1962) provides some information of the discriminability of transformed forms. Gibson, et.al., had children from ages four to eight choose forms in a match to sample recognition task where the alternatives included five classes of transformations of the sample forms. Four-year-old children had greatest difficulty distinguishing the sample figure from perspective transformations of the sample, but also made errors on rotational, reversal (reflectional) and line-to-curve transformations. None of the children had any difficulty discriminating "break-and-close" transformations. Eight-year-old children had difficulty only with perspective transformations. It seems reasonable to assume that if children have difficulty discriminating figures that have undergone various transformations, they would have an even greater problem basing their choices on these transformations.

Bijou and his collegues (1965) have been working on a series of programmed instruction experiments designed to influence the development of non-verbal concepts in normal and retarded children. Bijou has succeeded in teaching differential responses to rotated geometric forms with left and right orientation differences. Not surprisingly, he found the error rate decreasing monotonically with chronological age.

A perceptual study by Suppes & Bandy, reported in Suppes (1964), is closely related to the present investigation. The *Ss*, children ranging from three to twelve years old, were simply asked to point to which of two plane figures seemed most like a given standard figure. Each of the two figures was rotated or stretched a certain degree with respect to the standard figure. No correction procedure or other form of reinforcement was given. Roughly speaking, the results showed that *Ss* had a definite tendency to choose the figure with minimum rotation independent of the degree of stretch relative to the second figure that was a possible choice, but this tendency was not uniform. It was most pronounced in first graders and decreased in both age directions. Because no reinforcement was given, this study does not provide information on the relative difficulty of learning to recognize figures under a rotational or stretch transformation.

The present study was an attempt to establish the difficulty children have learning to distinguish three simple geometrical transformations from other similar transformations. The children were given minimal instructions and no pretraining.

Method

Subjects. Ten girls and fourteen boys were selected randomly from the fourth, fifth and sixth grades at the Grant School in Cupertino, California. The *Ss* ranged in age from nine to twelve years and from 100 to 120 in I.Q. None of the *Ss* had previously served in experiments of this type.

Stimuli. One hundred stimuli sheets for each of the three transformation conditions were used in the experiment. Each sheet contained a sample figure at the top (a three-sided, four-sided or five-sided unsymmetrical polygon with a horizontal baseline) which was separated from four response figures by a horizontal line (Figure 1). The response figures were arranged in two rows

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Insert Figure 1 about here

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of two figures to minimize position effects. One of the response figures was a transformation of the sample figure determined by the condition (rotation, reflection or stretch) and the remaining three response figures were additional transformations of the sample drawn from the eighteen listed in Table 1. The

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Insert Table 1 about here

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design of the stimulus sheets allowed the correct transformation to appear an equal number of times in the four response positions and approximately an equal number of times with each of the three types of polygons. In addition the eighteen irrelevant transformations occurred approximately an equal number of times with each type of figure and in the four response positions for each condition.

Conditions and experimental design. The three major experimental conditions corresponded to three simple geometrical transformations: 45-degree counterclockwise rotation, reflection about the horizontal axis and change to 2/3 size (stretch). Each S received all one hundred problems (trials) of a condition in one daily session about one-half hour in length and was run in the three conditions on consecutive days. A different randomization of the order of the stimulus sheets was used for each S in each condition. The randomizations were restricted allowing the correct figure to appear in a given position on no more than three consecutive trials and a given type of polygon to appear on no more than three consecutive trials.

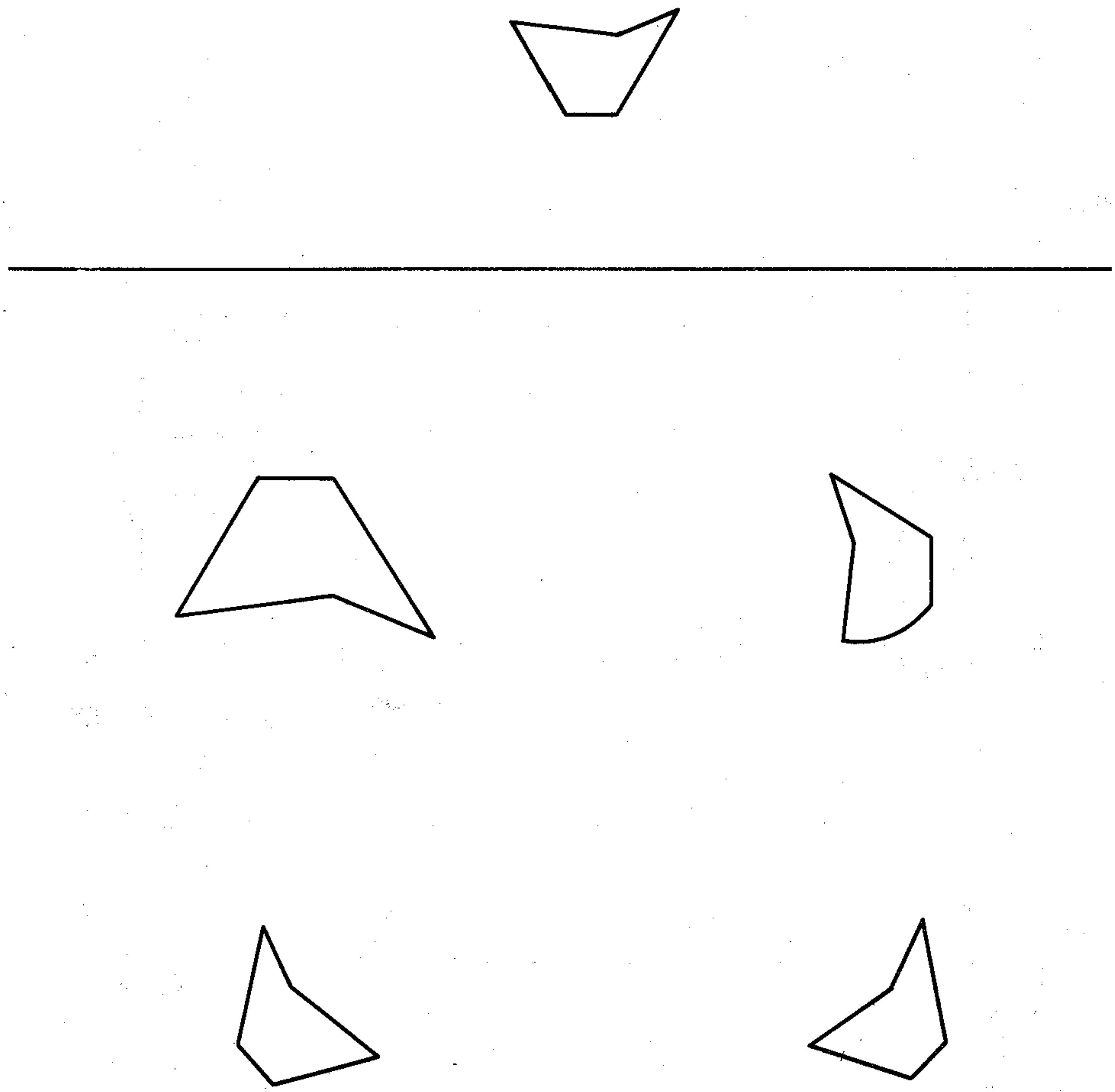

Figure 1. Sample stimulus sheet.

Table 1

Alternative Transformations

Only one transformation from each group may appear on a given stimulus sheet

Transformation Group	No.	Type	No.	Type
Curvature	1	convex	2	concave
Reflection & Rotation	3	horizontal & 45 degrees	4	vertical & 45 degrees
	5	horizontal & 90 degrees	6	vertical & 90 degrees
Rotation & Curvature	7	45 degrees & concave	8	45 degrees & convex
	9	90 degrees & concave	10	90 degrees & convex
Stretch & Curvature	11	2/3 size & concave	12	2/3 size & convex
	13	3/2 size & concave	14	3/2 size & convex
Stretch & Reflection	15	2/3 size & horizontal	16	2/3 size & vertical
	17	3/2 size & horizontal	18	3/2 size & vertical

Comments

Rotations are counterclockwise.

Reflections are mirror images with respect to the horizontal and vertical axes of the stimulus sheet. In rotation-reflection combinations the figure is first rotated and then reflected.

Stretch refers to changes in size of the length of the sides of the figures.

Curvature is applied to the side that forms an angle with the left edge of the baseline of the sample figure.

The Ss were divided into six experimental groups of four Ss each. The groups corresponded to the six orders that the three conditions could be presented in, and were balanced with respect to Ss' age, sex and grade level.

Procedure. The E brought the S to the experimental room and seated him at a desk. The E sat facing the S. At the first session the E gave the S the following instructions: "I am going to show you a group of sheets like this one". The E then uncovered a sample stimulus sheet. "The figures below the line are similar to the figure above the line, but differ from it in different ways. One of them is different in a certain correct way. You are to guess which figure differs from the top one in the correct way and point to it". The E then presented each of the one hundred sheets. The S was allowed as much time as needed to respond. If a correct response was made the E said "Good". If an incorrect response was made the E said "No", and then pointed to the correct figures and said, "It was this one". No additional instructions were given after the first session.

Results

Learning effects on conditions. Figure 2 presents the percentage of errors

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Insert Figure 2 about here

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per trial block averaged over all Ss for the three transformations. All three curves are positively accelerated and decreasing with a significant reduction of errors over trial blocks in all conditions ($F = 5.32$, $df = 9, 36$, $p < .01$). The total number of errors made on each of the three conditions also differed significantly ($F = 42.57$, $df = 2, 36$, $p < .01$). After the conclusion of the experiment a criterion of 15 consecutive correct responses was chosen to separate learners and non-learners. There were 10 learners in the reflection condition, 16 in the rotation condition and 24 in the stretch condition. Figure 3 presents the percentage of errors per trial block averaged over learners in the three

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Insert Figure 3 about here

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conditions. The results for learners are substantially the same as the results for all Ss. One unexpected finding was that first-trial performance was noticeably above chance ($p = .25$) for all conditions. The first-trial performance for the three conditions was 71% correct for rotation, 66% correct for reflection and 38% correct for stretch.

Learning effects on sessions. Figure 4 presents the percentage of errors

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Insert Figure 4 about here

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per trial block averaged over Ss for the three sessions. There was a significant decrease in total number of errors from the first to the last session ($F = 3.53$, $df = 2, 36$, $p < .01$).

Stationarity. Figure 5 presents Vincentized quartiles for the three conditions. Both the rotation and reflection conditions are stationary ($\chi^2 = .365$,

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Insert Figure 5 about here

$df = 9$, and $\chi^2 = 1.06$, $df = 9$) however, the stretch condition is nonstationary ($\chi^2 = 21.41$, $df = 9$, $p < .02$).

Effects of other variables. Tables 2 and 3 give the number of errors made

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Insert Tables 2 and 3 about here

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in each session by all Ss for each of the three types of polygons used as sample figures. There is no consistent difference in the number of errors made as a function of the number of sides of the sample figure.

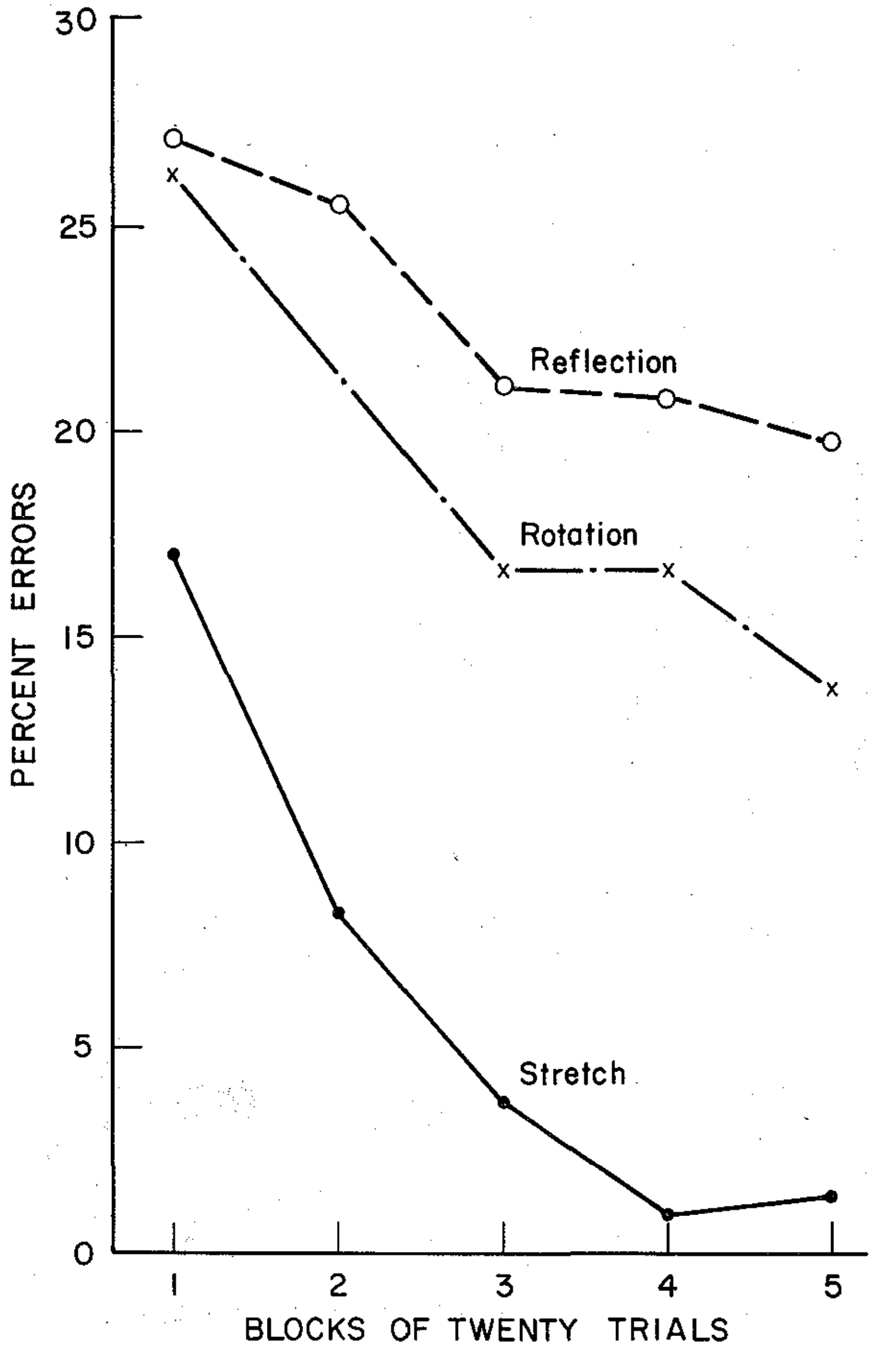

Figure 2. Percentage of Error for all Ss in the three experimental conditions.

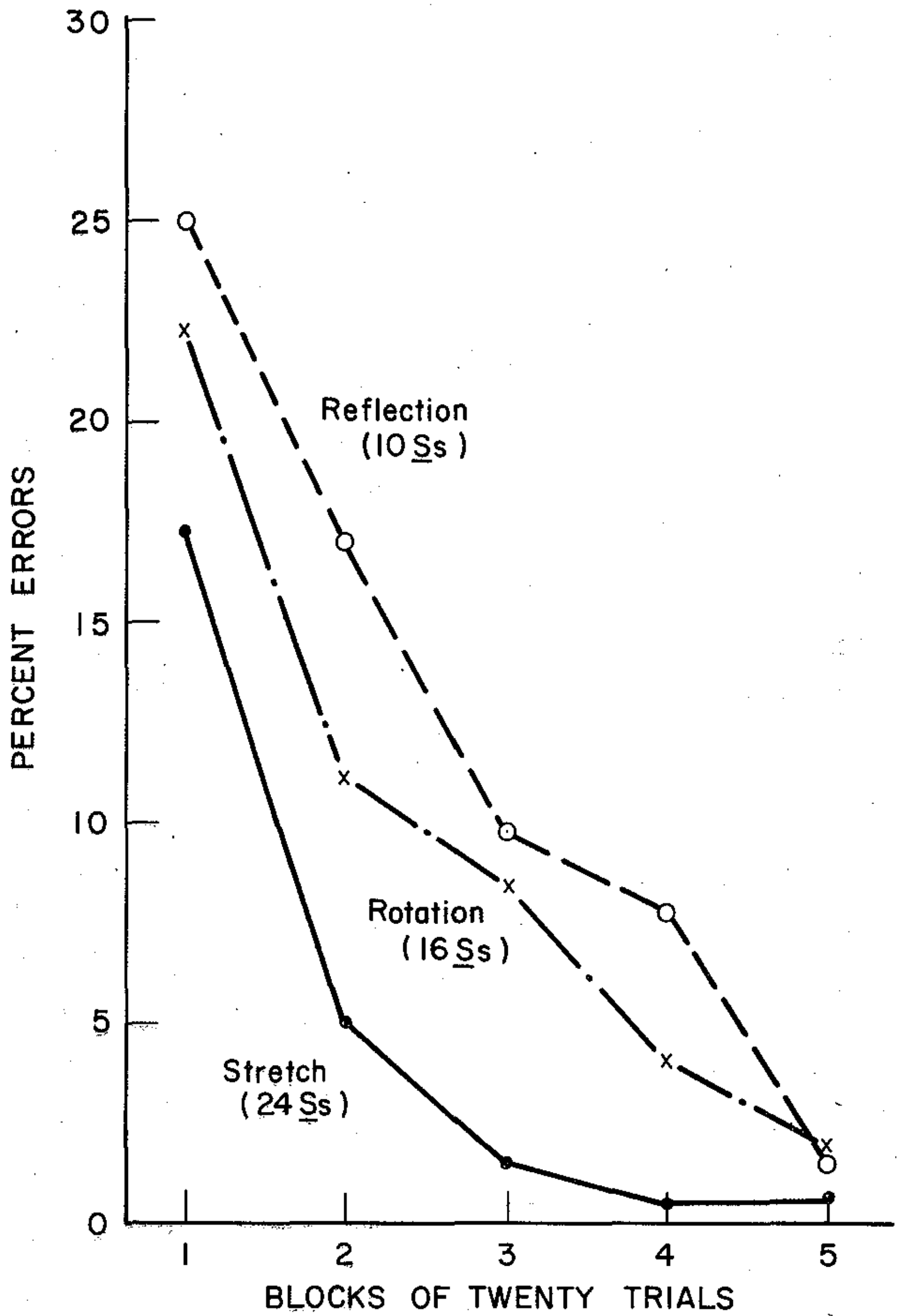

Figure 3. Percentage of errors for criterion Ss in the three experimental conditions.

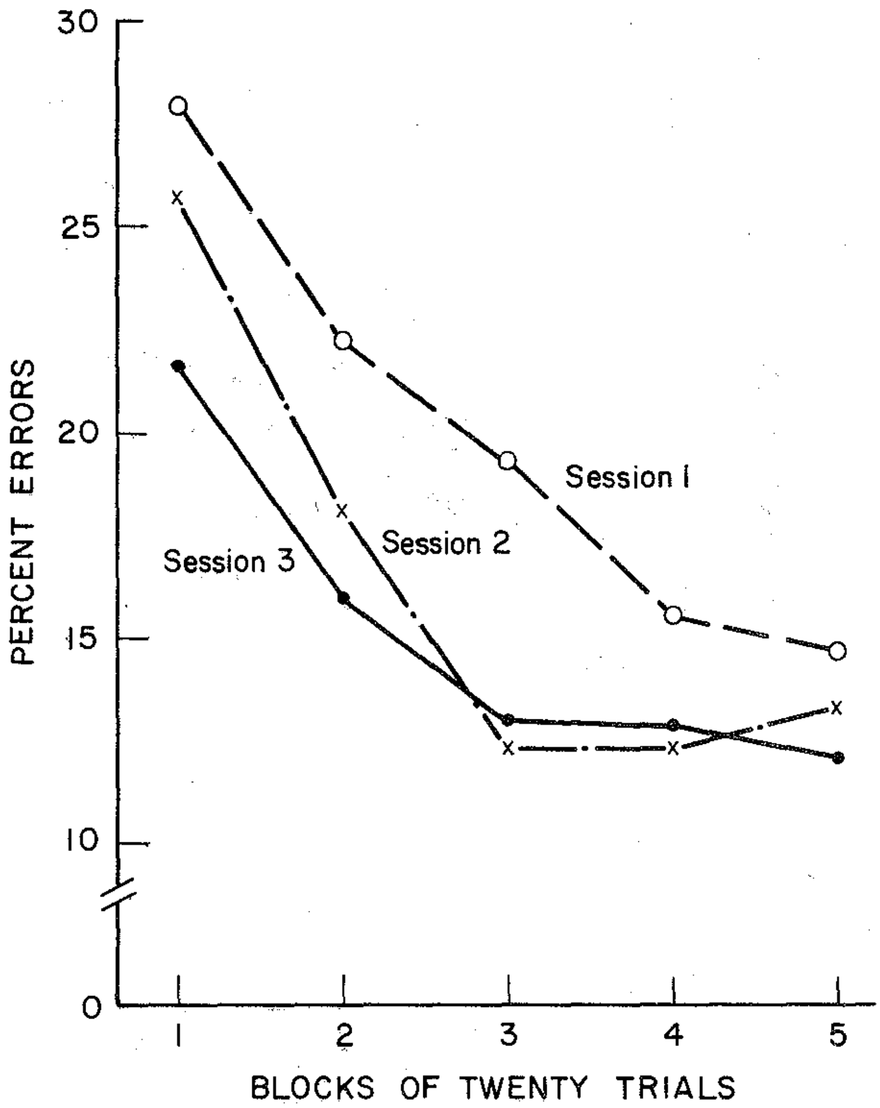

Figure 4. Percentage of errors for all Ss in the three experimental sessions.

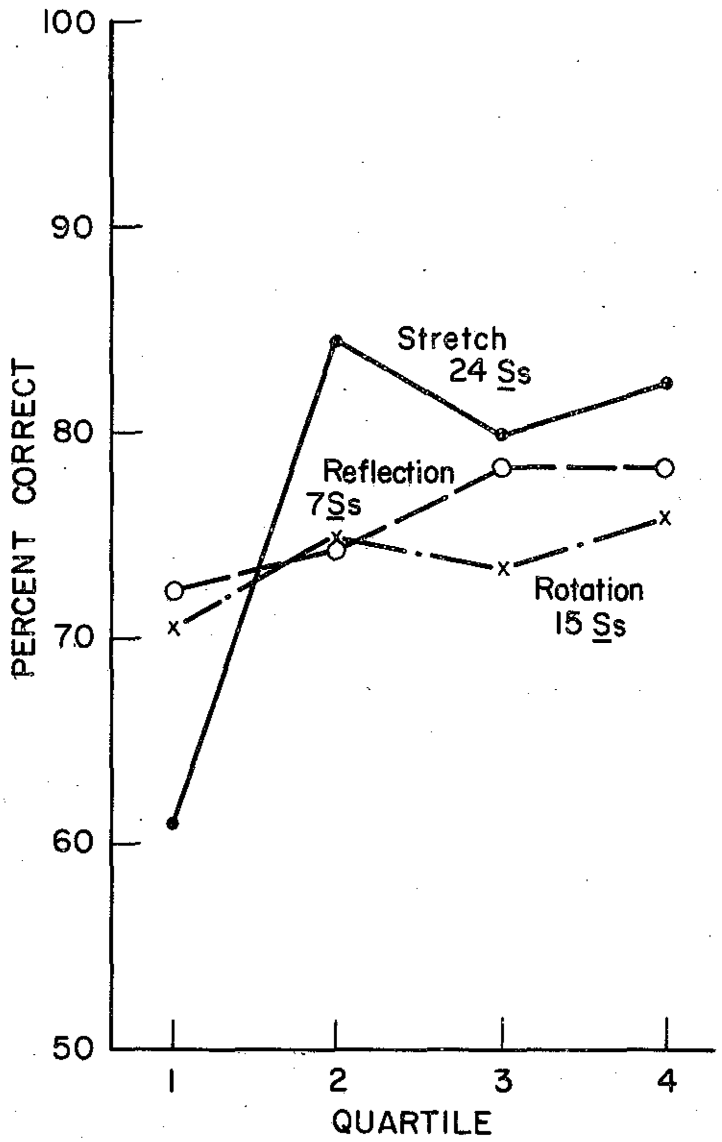

Figure 5. Vincent Quartile for criterion Ss in the three experimental conditions.

Table 2

Number of Errors Made on Figure Types

	Session			
Figure Type	1	2	3	Total
Triangle	160	128	110	398
Quadrilateral	146	130	119	395
Pentagon	162	128	117	407

Table 3

Number of Errors Made on Figure Types

	Condition		
Figure Type	Stretch	Rotation	Reflection
Triangle	68	156	174
Quadrilateral	47	156	192
Pentagon	44	154	209

There was a small but non-significant difference in the average number of errors made by males and females. The ten girls made an average of 48 errors and the fourteen boys made an average of 51 errors.

Discussion

The usually high percentage of correct responses on trial 1 of the rotation and reflection conditions as contrasted with trial-1 responses on the stretch condition suggested a closer scrutiny of the stimuli, particularly with respect to the sizes of the alternative response figures. Using the assumptions the Ss initially ignored response figures with a curved side and selected on the basis of size, the guessing probabilities on trial 1 should have been .72 for the rotation condition, .68 for the reflection condition and .42 for the stretch condition. The probabilities correspond very closely to the percentage of correct responses actually made on trial 1. The Ss appear to attend to size initially. This would explain in part why the stretch condition was simplest. The difference in difficulty between the rotation and reflection conditions is not explainable by such an apparently simple device. Children may find rotation a simpler concept or perhaps the difference found between rotation and reflection is an artifact of the magnitude of the rotation and the axis of reflection. Further research is underway to resolve this question.

The stationarity results indicate that both the rotation and reflection data might fit an all-or-none conditioning model (Suppes & Ginsberg, 1963). The stretch condition would not be fit by such a model. It is possible that the approximate stationarity of the rotation and reflection conditions is a reflection of the high initial guessing probabilities obtained in these conditions.

The Ss' improvement in performance as a function of sessions seems to be a learning-set effect. This may be attributable to some general increase in familiarity with the experimental situation, especially since there was no pretraining and minimal instructions were given.

There were insufficient data on each stimulus sheet (24 responses per sheet) to allow a careful analysis of the stimulus materials, however we intend to use the same stimuli in several experiments, including experiments where transfer of learning within type of transformation is possible. When each sheet has been responded to a large number of times a further analysis of the stimulus materials will be made. It is hoped that such an analysis will provide information on children's shape perception and will serve as an auxiliary to the concept learning data.

It is our own feeling that the fundamental theoretical problem for experiments like the one reported here is to provide a theory of learning that takes specific account of the perception of shapes and shape transformations. At a purely descriptive level, stimulus-sampling models of learning, with at most one or two postulated elements, can provide quite a good account of the course of learning for each of the transformations studied, but such models do not have the deeper structure required to make predictions about transfer or a priori predictions about the relative difficulty of different kinds of transformations. For perceptual problems as complex as the ones considered here, it is also apparent that we are a great distance from yet having neurological models of the brain that will yield such predictions. What does seem practical is the construction of models at an intermediary level of detail which postulates perceptual processing schema. In later papers dealing with the same stimuli we plan to pursue these matters at a theoretical level.

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Footnotes

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