A GENERALIZATION OF STIMULUS SAMPLING THEORY

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The phrase "Stimulus Sampling Theory" is used to describe various formulations of the basic theory first set forth by Estes [1950] and Estes and Burke [1953]. In this paper we shall restrict our attention to a particular set of axioms for Stimulus Sampling Theory; namely, the axioms given by Suppes and Atkinson [1960; Chapter 1]. The exact way in which these axioms deviate from the original Estes version is discussed by Suppes and Atkinson and will not be re-examined here; however, it should be emphasized that there is no deviation in basic ideas.

The purpose of this paper is to introduce what we consider to be a natural generalization of the axioms. The change leads to a set of axioms which, for special cases, is equivalent to the axioms in Suppes and Atkinson. The reason for introducing this modification is to provide a context in which such experimental variables as reward magnitude and motivation can be viewed as determiners of behavior. Further, some experimental results on multiple response problems have a natural interpretation in terms of the ideas presented in this paper.

We begin by stating the axioms for the two-response case since it is the simplest; the generalization to multiple responses will be examined later. As customary, the responses are denoted A_1 and A_2, and three reinforcing events E_0, E_1 and E_2 are specified.

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*/ On leave of absence from University of California, Los Angeles.

The first group of axioms deals with the conditioning of stimuli, the second group with the sampling of stimuli, and the third with responses.

Conditioning Axioms

C1. Associated with each stimulus element i is a positive integer s_i.

C2. At the start of trial n stimulus element i is in conditioning state $K_{j,n}$ where $j = 0, 1, 2, \ldots, s_i$.

C3. If stimulus element i is sampled on trial n and is in conditioning state $K_{j,n}$, then with probability $1 - \theta$ the reinforcing event is not effective and no change occurs in the conditioning state. When the reinforcing event is effective (i.e. with probability θ) then the conditioning state

(a) changes to K_{j+1} if E_1 occurs (however, if in $K_{s_i,n}$ then no change occurs),

(b) changes to K_{j-1} if E_2 occurs (however, if in $K_{0,n}$ then no change occurs),

(c) remains unchanged if E_0 occurs.

C4. Stimulus elements which are not sampled on a trial do not change their conditioning state on that trial.

C5. The probability θ is independent of the trial number and the preceding pattern of events.

Sampling Axioms

S1. *Exactly one stimulus element is sampled on each trial.*

S2. *Given the set of elements available for sampling on a trial, the probability of sampling a particular element is independent of the trial number and the preceding pattern of events.*

Response Axiom

R1. *If stimulus element* i *is in conditioning state* $K_{j,n}$ *and the element is sampled, then the probability of an* A_1 *response is* j/s_i.

These axioms are formally identical to those given by Suppes and Atkinson [1960] when $s_i = 1$ for all elements. For this case methods of estimating the number of elements (N) and the conditioning parameter θ have been worked out and many applications to empirical data are available.

When $s_i > 1$ for some elements, then interesting and rather surprising predictions occur. We now proceed to examine this case. In much of the discussion we shall restrict ourselves to the one-element model $(N = 1)$. There are no mathematical problems in extending the analysis to the multi-element case but notation becomes extremely complex. Further, a consideration of the one-element case is adequate for illustrating the basic ideas.

Noncontingent reinforcement. We begin with the simple noncontingent situation where E_0's are not permitted and the probability of events E_1 and E_2 are constant over trials; i.e., $P(E_{1,n}) = \pi \geq \frac{1}{2}$. We may prove from our axioms that the sequence of random variables which take the conditioning states as values is a Markov chain. This means, among other things, that a transition matrix $P = [p_{ij}]$ may be constructed where $p_{ij} = P(K_{j,n+1} | K_{i,n})$. The learning process is completely characterized by these transition probabilities and the initial probability distribution on the conditioning states.

By Axiom C3, it is obvious that

$$
\begin{aligned}
p_{s,s} &= 1 - \theta + \theta\pi \\
p_{s,s-1} &= \theta(1 - \pi)
\end{aligned}
$$

$$
(1) \qquad \left.\begin{aligned}
p_{i,i+1} &= \theta\pi \\
p_{i,i} &= 1 - \theta \\
p_{i,i-1} &= \theta(1 - \pi)
\end{aligned}\right\} \qquad i \neq 0,\ s
$$

$$
\begin{aligned}
p_{0,1} &= \theta\pi \\
p_{0,0} &= 1 - \theta + \theta(1 - \pi)
\end{aligned}
$$

Next define $p_{ij}^{(n)}$ as the probability of being in state j on trial n+1, given that on trial 1 we were in state i. Moreover, if the appropriate limit exists and is independent of i, we set

(2) $$u_j = \lim_{n \to \infty} p_{ij}^{(n)} .$$

The Markov chain defined by (1) is irreducible and aperiodic; for such a finite-state chain it is well known that the limiting quantities u_j exist. For our particular case

(3) $$u_j = \begin{cases} \dfrac{a^{s-j} - a^{s-j+1}}{1 - a^{s+1}} & \text{for} \quad a \neq 1 \\ \\ \dfrac{1}{s+1} & \text{for} \quad a = 1 \end{cases}$$

where $a = \frac{1 - \pi}{\pi}$.

By the Response Axiom R1 we have that the asymptotic probability of an A_1 response in the noncontingent situation is

$$\lim_{n \to \infty} P(A_{1,n}) = P(A_1) = \sum_{j=0}^{s} \frac{j}{s} u_j$$

(4) $$= \frac{s(1-a) - a(1-a^s)}{s(1-a)\ (1-a^{s+1})} \qquad \text{for} \quad \pi \neq \frac{1}{2}$$

$$= \frac{1}{2} \qquad \text{for} \quad \pi = \frac{1}{2}$$

For $\pi = \frac{1}{2}$ the prediction of $P(A_1)$ is $\frac{1}{2}$ for all values of s. However for $\pi \neq \frac{1}{2}$ the asymptotic prediction depends on s.

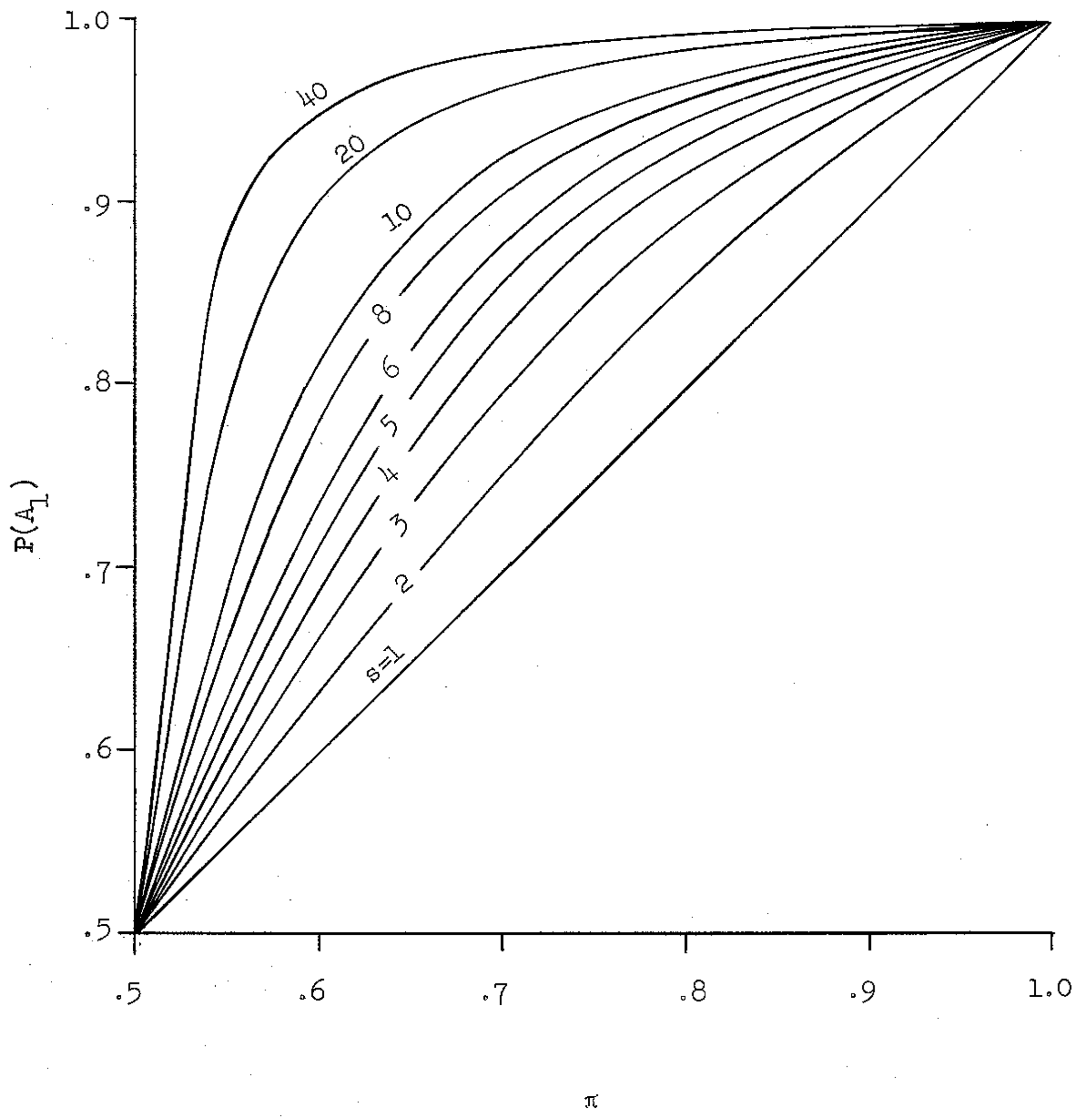

Figure 1. $P(A_1)$ as a function of π.

Figure 1 presents $P(A_1)$ as a function of π ; the parameter on each curve is the value of s. For s equal to 1 we have $P(A_1) = \pi$; however as s increases, the prediction for $P(A_1)$ becomes increasingly greater than π. In fact by inspection of (4) it is obvious that $\lim_{s \to \infty} P(A_1) = 1$ for $\pi > \frac{1}{2}$.*/

Suppes and Atkinson [1960, Chapter 10] report data for a non-contingent experiment where $\pi = .6$. The independent variable was the amount of money won or lost on each trial when the subject was correct $(A_{1,n}\ E_{1,n}$ or $A_{2,n}\ E_{2,n})$ or incorrect $(A_{2,n}\ E_{1,n}$ or $A_{1,n}\ E_{2,n})$. For subjects in Group Z, no money was won or lost; for Group F five cents was won when the subject was correct and the same amount lost when incorrect; for Group T ten cents was won or lost. The obtained proportions of A_1 responses at asymptote (trials 141-240) were .593 (Group Z), .644 (Group F) and .690 (Group T). If we were to estimate s for the one-element model from this data alone we would find that s is approximately 1.0 for Group Z, 2.3 for Group F, and 3.3 for Group T.

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*/ Comparable results can be obtained for other reinforcement schedules. For example, consider a contingent situation where E_0's are not permitted and let $P(E_{1,n}|A_{1,n}) = \pi_1$ and $P(E_{1,n}|A_{2,n}) = \pi_2$. For this case if $\frac{\pi_2}{1 - \pi_1 + \pi_2} > \frac{1}{2}$, then $P(A_1)$ approaches 1 as s becomes large. For example, if $\pi_1 = \frac{3}{4}$ and $\pi_2 = \frac{1}{2}$, then $P(A_1)$ is .67, .71, .75, .79, ... for s = 1, 2, 3, 4, ...

For this experiment the estimated value of s increased as a function of the monetary payoff. In terms of the elementary process the amount of change in response probability on a given trial is dependent on the monetary payoff. For example, in the one-element model if $P(A_{1,n}) = 0$, an E_1 occurs, and conditioning is effective then $P(A_{1,n+1}) = \frac{1}{s}$. Thus, the isolated effect of a single reinforcement is a function of the payoff.*/ Of course, these ideas apply directly to experimental situations where different amounts of money can be won or lost from trial to trial; more detailed notions concerning the relations of θ and s to monetary value will depend on this type of investigation.

These results on the one-element model can be extended to the multi-element case and thereby permit $P(A_1)$ to take any value in the interval $[\pi, 1)$. It should be noted that for $N > 1$ and any set of values for s_i $(i = 1, \ldots, N)$ we have a chain of infinite order in the sequence of response random variables; the same statement holds for $N = 1$ and $s > 1$. However, for the special case where $N = s = 1$, the sequence of response random variables is a first-order Markov chain (see Suppes and Atkinson [1960] for a discussion of this point).

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*/ An inspection of the entire set of data suggests that both θ and s increase as a function of monetary payoffs.

We shall not examine the multi-element problem but instead turn to some sequential results for the one-element noncontingent model. We present only a few to illustrate the method of proof and have selected those quantities which are useful in making pseudomaximum-likelihood estimates of θ and s. The reader is referred to Suppes and Atkinson [1960, Chapter 2] for a discussion of appropriate estimation procedures.

Consider first $P(A_{1,n+1} | E_{1,n}\ A_{1,n})$. By elementary probability considerations and Axiom R1 we have that

$$P(A_{1,n+1}\ E_{1,n}\ A_{1,n})$$

$$= \sum_{i,j} P(A_{1,n+1}\ K_{j,n+1}\ E_{1,n}\ A_{1,n}\ K_{i,n})$$

$$= \sum_{i,j} P(A_{1,n+1} | K_{j,n+1})\ P(K_{j,n+1} | E_{1,n} A_{1,n} K_{i,n})\ P(E_{1,n})\ P(A_{1,n} | K_{i,n})\ P(K_{i,n})$$

However, by Axiom C3 we have that

$$P(A_{1,n+1}\ E_{1,n}\ A_{1,n})$$

$$= \sum_{i=0}^{s-1} \left[\frac{i+1}{s}\ \theta + \frac{i}{s}\ (1-\theta) \right]\ \pi\ \frac{i}{s}\ P(K_{i,n}) + \pi\ P(K_{s,n})$$

$$= \frac{\theta\pi}{s} \sum_{i=0}^{s-1} \frac{i}{s}\ P(K_{i,n}) + \pi \sum_{i=0}^{s-1} \frac{i^2}{s^2}\ P(K_{i,n}) + \pi\ P(K_{s,n})$$

Note, however, that $\lim_{n \to \infty} P(K_{i,n}) = u_i$ and by (3) we have

$$\lim_{n \to \infty} P(A_{1,n+1}\ E_{1,n}\ A_{1,n}) = \frac{\theta\pi}{s}\,[P(A_1) - u_s] + \pi[V_2 - u_s] + \pi u_s$$

$$= \frac{\theta\pi}{s}\,[P(A_1) - u_s] + \pi V_2$$

where $V_2 = \sum_{i=0}^{s} (\frac{i}{s})^2 u_i$ and can be easily calculated. Thus

$$\lim_{n \to \infty} P(A_{1,n+1} | E_{1,n}\ A_{1,n}) = \frac{1}{P(A_1)}\,\{\frac{\theta}{s}\,[P(A_1) - u_s] + V_2\}$$

Other asymptotic predictions useful for estimating parameters may be obtained by similar arguments and are given below:

$$\lim_{n \to \infty} P(A_{1,n+1} | E_{2,n}\ A_{2,n}) = \frac{1}{P(A_2)}\,\{P(A_1) - V_2 + \frac{\theta}{s}\,[u_0 - P(A_2)]\}$$

$$\lim_{n \to \infty} P(A_{1,n+1} | E_{2,n}\ A_{1,n}) = \frac{V_2}{P(A_1)} - \frac{\theta}{s}$$

$$\lim_{n \to \infty} P(A_{1,n+1} | E_{1,n}\ A_{2,n}) = \frac{1}{P(A_2)}\,\{\frac{\theta}{s}\,P(A_2) + P(A_1) - V_2\}$$

$$\lim_{n \to \infty} P(A_{1,n+1} | E_{1,n}) = P(A_1) + \frac{\theta}{s}\,(1 - u_s)$$

$$\lim_{n \to \infty} P(A_{1,n+1} | E_{2,n}) = P(A_1) - \frac{\theta}{s}\,[1 - u_0]$$

$$\lim_{n \to \infty} P(A_{1,n+1} | A_{1,n}) = \frac{1}{P(A_1)}\left\{V_2 - P(A_1)\,\frac{\theta(1 - 2\pi)}{s} - \frac{u_s\theta\pi}{s}\right\}$$

$$\lim_{n \to \infty} P(A_{1,n+1} | A_{2,n}) = \frac{1}{P(A_2)}\left\{P(A_1) - V_2 + \frac{u_0\theta(1-\pi)}{s} - \frac{\theta(1-2\pi)}{s}\,P(A_2)\right\}$$

<u>Mean learning curves</u>. Expressions for mean learning curves generally can be obtained but the computations are often quite tedious. Consequently we shall not examine this topic in detail except to present results for the one-element noncontingent model when $\pi = 1$ and $P(K_{i,1}) = \frac{1}{s+1}$ for $i = 0, 1, \dots, s$. */

For $s = 1$, the transition matrix $P = [p_{ij}]$ is

$$\begin{array}{c|cc} & 1 & 0 \\ \hline 1 & 1 & 0 \\ 0 & \theta & 1-\theta \end{array}$$

and P^n is

$$\begin{array}{c|cc} & 1 & 0 \\ \hline 1 & 1 & 0 \\ 0 & 1-(1-\theta)^n & (1-\theta)^n \end{array}$$

Further, define $u_j^{(n)}$ as the probability of being in conditioning state j at the start of trial n (given a uniform distribution on trial 1). Then

$$u_1^{(n)} = u_1^{(1)} + u_0^{(1)} [1 - (1 - \theta)^{n-1}]$$

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*/ Gordon Bower has derived many results for the case where $s = 2$ and $P(K_{1,1}) = 1$ and is applying the model to paired-associate learning data (see forthcoming technical report).

But by Axiom R1

$$P(A_{1,n}) = u_1^{(n)} = 1 - \frac{1}{2}(1 - \theta)^{n-1}$$

Next consider the case where there are three conditioning states; i.e., where $s = 2$. The transition matrix is

$$\begin{array}{c|ccc} & 2 & 1 & 0 \\ \hline 2 & 1 & 0 & 0 \\ 1 & \theta & 1-\theta & 0 \\ 0 & 0 & \theta & 1-\theta \end{array}$$

and P^n is

$$\begin{array}{c|ccc} & 2 & 1 & 0 \\ \hline 2 & 1 & 0 & 0 \\ 1 & 1-(1-\theta)^n & (1-\theta)^n & 0 \\ 0 & 1-(1-\theta)^n - n\theta(1-\theta)^{n-1} & n\theta(1-\theta)^{n-1} & (1-\theta)^n \end{array}$$

Then

$$u_2^{(n)} = u_2^{(1)} + u_1^{(1)}[1-(1-\theta)^{n-1}] + u_0^{(1)}[1-(1-\theta)^{n-1} - (n-1)\ \theta(1-\theta)^{n-2}]$$

$$u_1^{(n)} = u_1^{(1)}(1-\theta)^{n-1} + u_0^{(1)}(n-1)\ \theta(1-\theta)^{n-2}$$

And by Axiom R1

$$P(A_{1,n}) = u_2^{(n)} + \frac{1}{2} u_1^{(n)} = 1 - \frac{1}{2} (1-\theta)^{n-1} - \frac{1}{6} (n-1)\ \theta(1-\theta)^{n-2}$$

for $n \geq 3$. $P(A_{1,2}) = \frac{1}{2} + \frac{1}{3}\theta$ and of course $P(A_{1,1}) = \frac{1}{2}$. For $s = 3$, the transition matrix is

$$\begin{array}{c|cccc} & 3 & 2 & 1 & 0 \\ \hline 3 & 1 & 0 & 0 & 0 \\ 2 & \theta & 1-\theta & 0 & 0 \\ 1 & 0 & \theta & 1-\theta & 0 \\ 0 & 0 & 0 & \theta & 1-\theta \end{array}$$

and P^n is

$$\begin{array}{c|cccc} & 3 & 2 & 1 & 0 \\ \hline 3 & 1 & 0 & 0 & 0 \\ 2 & 1-(1-\theta)^n & (1-\theta)^n & 0 & 0 \\ 1 & 1-(1-\theta)^n - n\theta(1-\theta)^{n-1} & n\theta(1-\theta)^{n-1} & (1-\theta)^n & 0 \\ 0 & 1-(1-\theta)^n - n\theta(1-\theta)^{n-1} - \binom{n}{2}\ \theta^2(1-\theta)^{n-2} & \binom{n}{2}\ \theta^2(1-\theta)^{n-2} & n\theta(1-\theta)^{n-1} & (1-\theta)^n \end{array}$$

Then

$$u_3^{(n)} = u_3^{(1)} + u_2^{(1)}[1-(1-\theta)^{n-1}] + u_1^{(1)}[1-(1-\theta)^{n-1} - (n-1)\ \theta(1-\theta)^{n-2}]$$

$$+ u_0^{(1)}[1-(1-\theta)^{n-1} - (n-1)\ \theta(1-\theta)^{n-2} - \binom{n-1}{2}\ \theta^2(1-\theta)^{n-3}]$$

$$u_2^{(n)} = u_2^{(1)}(1-\theta)^{n-1} + u_1^{(1)}(n-1)\ \theta(1-\theta)^{n-2} + u_0^{(1)}\binom{n-1}{2}\ \theta^2(1-\theta)^{n-3}$$

$$u_1^{(n)} = u_1^{(1)}(1-\theta)^{n-1} + u_0^{(1)}(n-1)\ \theta(1-\theta)^{n-2}$$

And by Axiom R1

$$P(A_{1,n}) = u_3^{(n)} + \frac{2}{3}u_2^{(n)} + \frac{1}{3}u_1^{(n)}$$

$$= 1 - \frac{1}{2}(1-\theta)^{n-1} - \frac{1}{4}(n-1)\ \theta(1-\theta)^{n-2} - \frac{1}{12}\binom{n-1}{2}\ \theta^2(1-\theta)^{n-3}$$

for $n \geq 4$. $P(A_{1,n}) = \frac{1}{2}$, $P(A_{2,n}) = \frac{1}{2} + \frac{1}{4}\theta$, and $P(A_{3,n}) = \frac{5}{12} + \frac{1}{2}\theta + \frac{1}{12}(1-\theta)(1+\theta)$. We shall not pursue the general case, although it is obvious that

$$P(A_{1,n}) = 1 - c_1(1-\theta)^{n-1} - c_2(n-1)\ \theta(1-\theta)^{n-2} - \cdots - c_s\binom{n-1}{s-1}\ \theta^{s-1}(1-\theta)^{n-s}$$

where $0 < c_j \leq \frac{1}{2}$.

Thus, the value of s affects not only the rate of learning but also the form of the learning curve. With $s = 1$ we have the standard exponential growth function, but as s becomes large the form of the curve becomes $\int$-shaped.

Multiple Responses. We now examine the case where there are r responses $(A_1, \ldots, A_r)$ and $r+1$ reinforcing events $(E_0, E_1, \ldots, E_r)$. For the multiple response case it is necessary to restate axioms C2, C3 and R1 more generally.

C2'. At the start of trial n stimulus element i is in conditioning state $<k_{1,n}\ k_{2,n} \cdots k_{r,n}>$ where $k_{j,n} = 0, 1, \ldots, s_i$ and $k_{1,n} + k_{2,n} + \cdots + k_{r,n} = s_i$.

C3'. If stimulus element i is sampled on trial n and is in conditioning state $<k_{1,n} \cdots k_{r,n}>$, then with probability $1-\theta$ the reinforcing event is not effective and no change occurs in the conditioning state. When the reinforcing event is effective (i.e. with probability θ)

(a) if $E_{\ell,n}$ $(\ell \neq 0)$ occurs, then $k_{\ell,n+1} = k_{\ell,n} + 1$ and one and only one of the other k's takes a decrement of 1. The probability (for $j \neq \ell$) that $k_{j,n+1} = k_{j,n} - 1$ is $k_{j,n}/(s_i - k_{\ell,n})$

(b) if $E_{0,n}$ occurs, then the conditioning state remains unchanged.

R1'. If stimulus element i is in conditioning state $<k_{1,n} \cdots k_{r,n}>$ and the element is sampled, then the probability of response A_j is $k_{j,n}/s_i$.

For $r = 2$ these axioms are equivalent to the axioms given at the outset of this paper. The only reason for introducing the earlier version was to make the presentation of the two-response case more accessible.

We now apply the axioms to a noncontingent reinforcement procedure reported by Gardner [1957]. Three responses (A_1, A_2, A_3) are available to the subject and three reinforcing events (E_1, E_2, E_3) are employed. On each trial one of the reinforcing events occurs; i.e., $P(E_{i,n}) = \pi_i$ where $\pi_1 + \pi_2 + \pi_3 = 1$. Again, we consider only the one-element case, but there are no mathematical problems in extending this analysis to multiple elements; the only difficulty is that notation and computations can become very involved.

First consider the case where $s = 1$. There are three conditioning states $\langle 100 \rangle$, $\langle 010 \rangle$ and $\langle 001 \rangle$. These states form a Markov chain whose transition matrix can be obtained from Axiom C3' and is as follows:

$$
\begin{array}{c|ccc}
 & \langle 100 \rangle & \langle 010 \rangle & \langle 001 \rangle \\
\hline
\langle 100 \rangle & 1-\theta+\theta\pi_1 & \theta\pi_2 & \theta\pi_3 \\
\langle 010 \rangle & \theta\pi_1 & 1-\theta+\theta\pi_2 & \theta\pi_3 \\
\langle 001 \rangle & \theta\pi_1 & \theta\pi_2 & 1-\theta+\theta\pi_3
\end{array}
$$

Define u_{ijk} $(i, j, k = 1, 0)$ analogous to (2). Then by Axiom R1'

(5)
$$\lim_{n \to \infty} P(A_{1,n}) = P(A_1) = u_{100} = \pi_1$$

$$\lim_{n \to \infty} P(A_{2,n}) = P(A_2) = u_{010} = \pi_2$$

$$\lim_{n \to \infty} P(A_{3,n}) = P(A_3) = u_{001} = \pi_3$$

For $s = 2$, the conditioning states are $\langle 200\rangle$, $\langle 110\rangle$, $\langle 101\rangle$, $\langle 020\rangle$, $\langle 011\rangle$, $\langle 002\rangle$ and the transition matrix is as follows:

	$\langle 200\rangle$	$\langle 110\rangle$	$\langle 101\rangle$	$\langle 020\rangle$	$\langle 011\rangle$	$\langle 002\rangle$
$\langle 200\rangle$	$1-\theta+\theta\pi_1$	$\theta\pi_2$	$\theta\pi_3$			
$\langle 110\rangle$	$\theta\pi_1$	$1-\theta$	$\frac{1}{2}\theta\pi_3$	$\theta\pi_2$	$\frac{1}{2}\theta\pi_3$	
$\langle 101\rangle$	$\theta\pi_1$	$\frac{1}{2}\theta\pi_2$	$1-\theta$		$\frac{1}{2}\theta\pi_2$	$\theta\pi_3$
$\langle 020\rangle$		$\theta\pi_1$		$1-\theta+\theta\pi_2$	$\theta\pi_3$	
$\langle 011\rangle$		$\frac{1}{2}\theta\pi_1$	$\frac{1}{2}\theta\pi_1$	$\theta\pi_2$	$1-\theta$	$\theta\pi_3$
$\langle 002\rangle$			$\theta\pi_1$		$\theta\pi_2$	$1-\theta+\theta\pi_3$

It can be shown that

$$u_{200} = \pi_1^2/A \qquad u_{020} = \pi_2^2/A$$

$$u_{110} = \pi_1\pi_2/A \qquad u_{011} = \pi_2\pi_3/A$$

$$u_{101} = \pi_1\pi_3/A \qquad u_{002} = \pi_3^2/A$$

where $A = \pi_1^2 + \pi_2^2 + \pi_3^2 + \pi_1\pi_2 + \pi_1\pi_3 + \pi_2\pi_3$.

By Axiom R1'

$$
\begin{aligned}
P(A_1) &= u_{200} + \tfrac{1}{2}\,[u_{110} + u_{101}] = \pi_1[\pi_1 + \tfrac{1}{2}\,(1-\pi_1)]/A \\
P(A_2) &= u_{020} + \tfrac{1}{2}\,[u_{110} + u_{011}] = \pi_2[\pi_2 + \tfrac{1}{2}\,(1-\pi_2)]/A \qquad (6) \\
P(A_3) &= u_{002} + \tfrac{1}{2}\,[u_{101} + u_{011}] = \pi_3[\pi_3 + \tfrac{1}{2}\,(1-\pi_3)]/A
\end{aligned}
$$

For $s = 3$ there are 10 conditioning states and the transition matrix is as follows:

	$\langle 300\rangle$	$\langle 210\rangle$	$\langle 201\rangle$	$\langle 120\rangle$	$\langle 111\rangle$	$\langle 030\rangle$	$\langle 021\rangle$	$\langle 102\rangle$	$\langle 012\rangle$	$\langle 003\rangle$
$\langle 300\rangle$	$1-\theta+\theta\pi_1$	$\theta\pi_2$	$\theta\pi_3$							
$\langle 210\rangle$	$\theta\pi_1$	$1-\theta$	$\frac{1}{3}\,\theta\pi_3$	$\theta\pi_2$	$\frac{2}{3}\,\theta\pi_3$					
$\langle 201\rangle$	$\theta\pi_1$	$\frac{1}{3}\,\theta\pi_2$	$1-\theta$		$\frac{2}{3}\,\theta\pi_2$			$\theta\pi_3$		
$\langle 120\rangle$		$\theta\pi_1$		$1-\theta$	$\frac{2}{3}\,\theta\pi_3$	$\theta\pi_2$	$\frac{1}{3}\,\theta\pi_3$			
$\langle 111\rangle$		$\frac{1}{2}\,\theta\pi_1$	$\frac{1}{2}\,\theta\pi_1$	$\frac{1}{2}\,\theta\pi_2$	$1-\theta$		$\frac{1}{2}\,\theta\pi_2$	$\frac{1}{2}\,\theta\pi_3$	$\frac{1}{2}\,\theta\pi_3$	
$\langle 030\rangle$				$\theta\pi_1$		$1-\theta+\theta\pi_2$	$\theta\pi_3$			
$\langle 021\rangle$				$\frac{1}{3}\,\theta\pi_1$	$\frac{2}{3}\,\theta\pi_1$	$\theta\pi_2$	$1-\theta$		$\theta\pi_3$	
$\langle 102\rangle$			$\theta\pi_1$		$\frac{2}{3}\,\theta\pi_2$			$1-\theta$	$\frac{1}{3}\,\theta\pi_2$	$\theta\pi_3$
$\langle 012\rangle$					$\frac{2}{3}\,\theta\pi_1$		$\theta\pi_2$	$\frac{1}{3}\,\theta\pi_1$	$1-\theta$	$\theta\pi_3$
$\langle 003\rangle$								$\theta\pi_1$	$\theta\pi_2$	$1-\theta+\theta\pi_3$

And by Axiom R1'

$$
\begin{aligned}
P(A_1) &= u_{300} + \tfrac{2}{3}\,[u_{210} + u_{201}] + \tfrac{1}{3}\,[u_{120} + u_{110} + u_{102}] \\
P(A_2) &= u_{030} + \tfrac{2}{3}\,[u_{120} + u_{021}] + \tfrac{1}{3}\,[u_{210} + u_{111} + u_{012}] \qquad (7) \\
P(A_3) &= u_{003} + \tfrac{2}{3}\,[u_{102} + u_{012}] + \tfrac{1}{3}\,[u_{201} + u_{111} + u_{021}]
\end{aligned}
$$

The analysis may be extended to any value of s. For r responses the number of conditioning states will be $\binom{r+s-1}{s}$. However, for our examination of the Gardner data a comparison of predictions for s equal to 1, 2, and 3 will be sufficient.

Gardner actually reports several experiments, but we shall consider only the data of Experiment I. Six groups were run. Two groups employed responses A_1 and A_2 and reinforcing events E_1 and E_2. The groups were denoted (70-30) and (60-40); the first number indicates the value of π, and the second the value of $1-\pi$. Asymptotic predictions for these groups are given by (4). The other groups involved three responses and were denoted (70-15-15), (70-20-10), (60-20-20) and (60-30-10); the first number indicates the value of π_1, the second the value of π_2, and the third the value of π_3. Asymptotic predictions for these groups are given by (5) for s equal to 1, by (6) for s equal to 2, and by (7) for s equal to 3.

The predicted values for s equal to 1 and 2 are presented in Table 1 along with Gardner's observed proportions on trails 286-450.

TABLE 1

PREDICTED AND OBSERVED ASYMPTOTIC PROPORTIONS FOR THE GARDNER DATA

Group	$P(A_1)$			$P(A_2)$			$P(A_3)$		
	Obs.	Predicted		Obs.	Predicted		Obs.	Predicted	
		s=1	s=2		s=1	s=2		s=1	s=2
60-40	.618	.600	.631	.382	.400	.369	---	---	---
60-30-10	.684	.600	.658	.235	.300	.267	.081	.100	.075
60-20-20	.676	.600	.667	.162	.200	.166	.162	.200	.166
70-30	.721	.700	.753	.279	.300	.279	---	---	---
70-20-10	.798	.700	.773	.129	.200	.156	.073	.100	.071
70-15-15	.802	.700	.800	.099	.150	.100	.099	.150	.100

Over-all, the predictions for $s = 2$ give a fairly good account of the data. However, for comparable experimental procedures and equipment, one would hope that the number of response alternatives would not affect the estimated value of s. Unfortunately this invariance in s is not perfectly reflected in these data. For example, the predicted value of $P(A_1)$ for $s = 2$ is slightly low for the two-response groups and somewhat high for the three-response groups. Of course, this could be a statistical artifact, and a satisfactory answer would depend on a more detailed analysis of the sequential data.

There are several general comments to be made concerning these predictions. First of all, for s greater than 1 the predicted value of $P(A_1)$ in the (70-30) group is less than the predicted value of $P(A_1)$ for groups (70-15-15) and (70-20-10); similarly, the predicted value of $P(A_1)$ for the (60-40) group is less than $P(A_1)$ for groups (60-20-20) and (60-30-10). This result holds in general for the noncontingent reinforcement model: if the A_1 response is reinforced with some specified probability greater than $\frac{1}{2}$, then for a fixed s greater than 1, the prediction for $P(A_1)$ increases as a function of the number of alternative responses. Further, $P(A_1)$ approaches 1 as s becomes large, independent of the number of alternative responses.

Another result can be established for the three-response noncontingent model. Let $\pi_1 > \frac{1}{2}$, $\pi_2 \geq \pi_3$, and define $\delta = \pi_2 - \pi_3$. Then we can prove for fixed values of π_1 and s (where $s > 1$) that $P(A_1)$ increases as δ approaches 0.

We shall not go further in our analysis of these axioms; our purpose in this paper has been simply to display the modified set of axioms and outline some of the grosser implications. Currently we are carrying out a detailed evaluation of the axioms with regard to several sets of data; future explorations of the ideas presented in this paper will depend on the success of these analyses.

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