THE USE OF MODELS IN EXPERIMENTAL PSYCHOLOGY

by

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THE USE OF MODELS IN EXPERIMENTAL PSYCHOLOGY*/

by

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In this paper I shall not be concerned with a formal analysis of the function of models in psychology. The problem has been considered on many occasions by both psychologists and philosophers, and I am not inclined to add to the voluminous literature in this area. Instead, I shall describe a fairly simple model of behavior and illustrate the method of application to a complex problem in decision making. By examination of this particular case we will be able to indicate the role of mathematical models in determining programs of psychological research and specifying the types of empirical observations to be made.

The case to be examined deals with the psychology of learning. There are three basic concepts in this area which play a central role in both theoretical and experimental work; they are the concepts of stimulus, response, and reinforcement. The stimulus is conceived as an environmental event; a response is an act or movement made by the organism exposed to the stimulating situation; and a reinforcement is any event (experimenter or subject controlled) which gives rise to an increment or decrement in the

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likelihood that a stimulus will elicit a particular response. One of the principal problems of learning psychologists is to specify the relations between stimuli and various response measures as a function of reinforcement schedules. A major contribution of early behavioristic scientists like Pavlov, Thorndike, Bechterev, Watson, and Guthrie was the development of these concepts in a scientific sense, clearly disentangling them from notions of common sense and of earlier philosophical psychology.

The introduction of mathematical formulations as a tool in the analysis of learning has been of fairly recent origin. Some of the earlier work by Schükarew, Robertson, Thurstone, Woodrow and others was concerned with finding an analytic function which was to provide a universal description of learning. Many suggestions for the appropriate function were made and there was much debate as to which was correct. This search for the universal learning function began with *ad hoc* proposals and curve fitting to experimental data. Gradually, however, these endeavors gave way to the more constructive undertaking of providing systematic formulations of the elementary events underlying the learning process.

One of the most noteworthy programs in constructing a quantitative theory of learning was that of Clark L. Hull. In his *Principles of Behavior* he presents a set of postulates designed to encompass the major aspects of learning. From these postulates deductions were made which initiated a great deal of empirical research. Unfortunately, Hull's work, and that of his contemporaries like Tolman and Lewin, did not lead to a theory that was mathematically viable. That is, in Hull's system it is possible to make only a very limited number of derivations leading to new quantitative predictions of behavior.

However, the work of Hull, Tolman, Lewin and others emphasized the importance of rigorous theory construction in psychology and set the stage for recent developments in mathematical learning theory. The work by Estes [1950], Bush and Mosteller [1950] and Estes and Burke [1953] initiated these new developments and represented analyses of learning which led to mathematically tractable systems. The work of these investigators and subsequent work of Luce, Suppes, Restle, Audley, and many others has resulted in systematic formulations of learning which have the same sort of feel about them that theories in physics have. Nontrivial quantitative predictions can be made--not only about the gross phenomena of learning but also with regard to the fine structure of the data. Once appropriate identification of theoretical terms has been made it is usually clear how to derive predictions about responses in a manner that is not ad hoc and is mathematically exact.

To illustrate some of these points I would like to present a particular set of axioms for describing learning. Only those axioms will be presented that are necessary for the analysis of the experiment to be considered in this paper. The reader interested in a more comprehensive formulation is referred to Suppes and Atkinson [1960].

The experimental situation consists of a sequence of discrete trials. There are K response alternatives, denoted $A_i (i = 1, \ldots, K)$. On each trial of the experiment two or more alternatives are made available to the subject, and he is required to select one of the available responses. Once his response has been made the subject wins or loses a fixed amount of money. The subject's task is to win as frequently as possible. There are many aspects of the situation that can be manipulated by the experimenter but in this paper we will consider only the following variables: (1) the strategy by which the experimenter makes available certain subsets of responses on any trial of the experiment, (2) the schedule by which the experimenter determines whether the occurrence of a particular response by the subject leads to a win or loss and (3) the amount of money won or lost on each trial. The role of the model in this situation is to provide an explicit and detailed account of a subject's responses over trials of the experiment. One reason for investigating this particular experimental problem is that it is a prototype of many decision making situations in the real world. If behavior can be predicted with accuracy in our laboratory situation, then we shall have substantially increased our understanding of decision processes in general.

The model we shall consider assumes that (1) associated with each response alternative there is a tendency to approach or avoid that alternative and (2) the response which is finally made on a trial depends on the observing or orienting behavior of the subject in the pre-decision period of the trial. The basic notions underlying the model are similar to those presented by Bower [1959], Estes [1960] and Audley [1960].

The axioms will be formulated verbally. It is not difficult to state them in a mathematically exact form, but for our purposes this will not be necessary.

A1. On every trial each response has an approach-avoidance value (AAV) of 1 or 0 .

A2. At the start of each trial the subject randomly observes one of the available responses.

A3. If the AAV for a particular available response is 1 and the response is observed, then that response will be made. If the AAV is 0 and the response is observed, then the subject will randomly reorient and observe one of the other available response alternatives.

A4. If all available responses have been observed on a trial and no response has been selected (i.e., the case where all available responses have AAV's of 0), then the subject terminates the trial by randomly selecting one of the available responses.

A5. If a response is selected on a trial and followed by a win, then with probability ρ' its AAV becomes 1 and with probability $1-\rho'$ its value remains unchanged. If a response is selected and followed by a loss, then with probability ρ'' its AAV becomes 0 and with probability $1-\rho''$ its value remains unchanged.*/ The reinforcement parameters ρ' and ρ'' are independent of the trial number and the preceding pattern of events.

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*/ We presume that ρ' and ρ'' are monotone increasing functions of the amount of money that can be won or lost on a trial.

A6. <u>The AAV associated with a response not selected on a given trial does not change on that trial</u>.

Several experiments have been conducted to test the adequacy of these axioms, but we shall restrict ourselves to one reported by Suppes and Atkinson [1960]. Subjects were run for 360 trials and on every trial they won or lost a fixed amount of money. There were four responses (A_1, A_2, A_3 and A_4) and on each trial exactly two of these responses were made available to the subject; the six possible response pairs occurred with equiprobability. On each trial the subject was required to select between the two available responses but was given no other information. A win or loss on a trial depended on the response selected. If A_i was available and chosen, then with probability ξ_i the subject won and with probability $1 - \xi_i$ he lost. The values used were as follows: $\xi_1 = .2$, $\xi_2 = .4$, $\xi_3 = .6$ and $\xi_4 = .8$.*/ Thus, if a subject is to maximize his probability of a win, he should choose A_4 whenever it is available, A_3 when it is available and A_4 is not available, and finally A_2 if neither A_3 nor A_4 is available.

For mathematical analysis in the remainder of this paper it will be useful to introduce the following notation:

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*/ A quite different model for this situation has been proposed by Suppes [1959].

$D_n^{(ij)}$ = the experimenter controlled event of making response pair (A_iA_j) available on trial n of the experiment $(i \neq j)$.

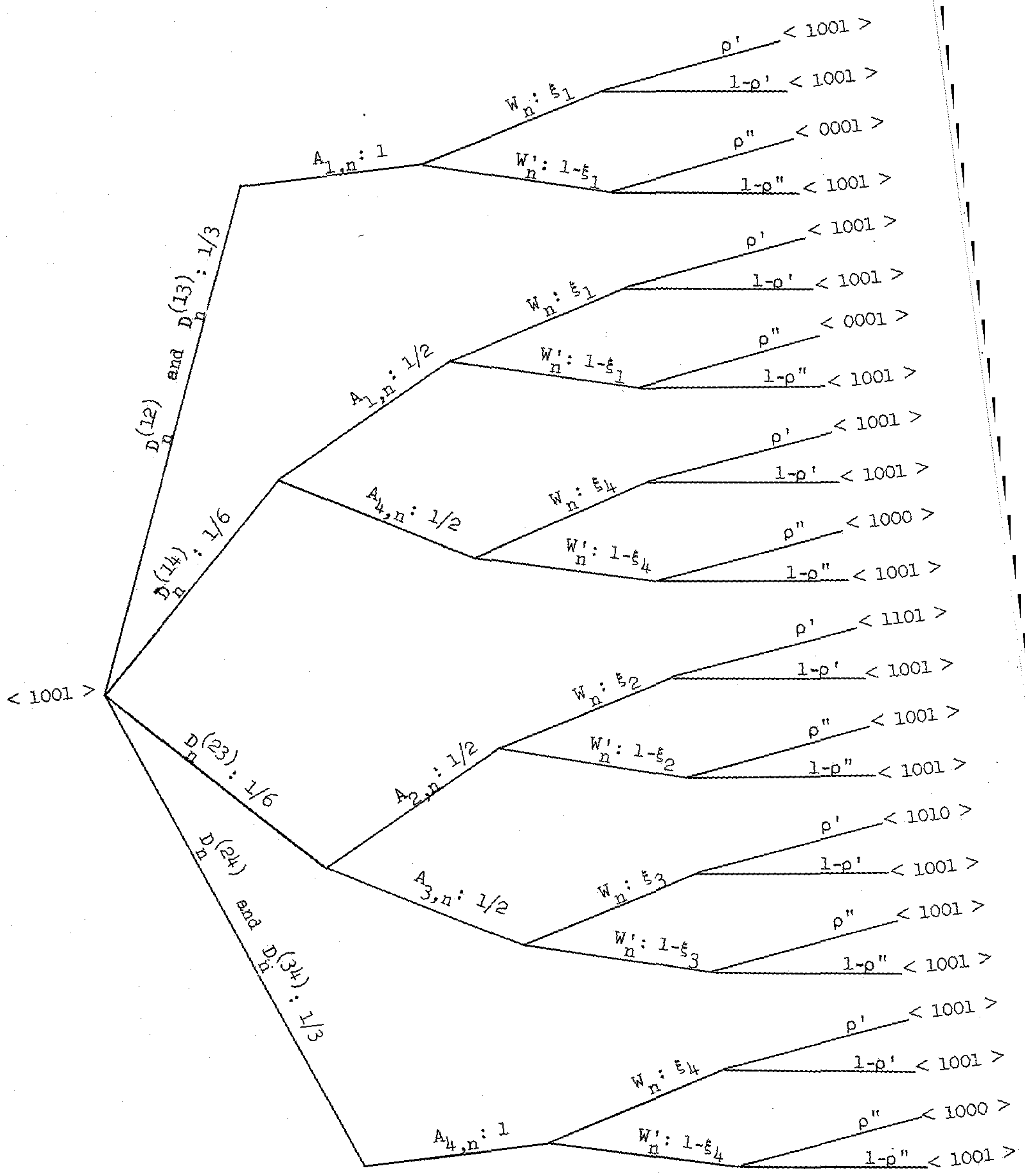
$A_{i,n}$ = selection by the subject of response A_i on trial n .

W_n = a win on trial n .

W'_n = a loss on trial n .

In terms of the axioms we define the subject-state on any trial of the experiment by an ordered four-tuple $< ijk\ell >$ where $i,j,k,\ell = 1$ or 0 . The first entry denotes the AAV assigned to response A_1 , the second the value for A_2 , and so on. From the axioms it can be shown that, for our particular experimental procedure, the sequence of random variables which take the subject-states as values is a Markov chain. This means, among other things, that a transition matrix $P = [p_{ij}]$ may be constructed where p_{ij} is the probability of being in subject-state j on trial $n+1$ given subject-state i on trial n . The learning process is completely characterized by these transition probabilities and the initial probability distribution on the states.

To illustrate the application of our axioms, we will derive one row of the transition matrix. In making such a derivation it is convenient to represent the various possible occurrences on a trial by a tree. Assume that we are in state $< 1001 >$ on trial n , then the appropriate tree is given in Figure 1. As indicated on the top branch, when the response pair



< 1001 >
$D_n^{(12)}$ and $D_n^{(13)}$: 1/3
$D_n^{(14)}$: 1/6
$D_n^{(23)}$: 1/6
$D_n^{(24)}$ and $D_n^{(34)}$: 1/3
$A_{1,n}$: 1
$A_{1,n}$: 1/2
$A_{4,n}$: 1/2
$A_{2,n}$: 1/2
$A_{3,n}$: 1/2
$A_{4,n}$: 1
W_n: ξ_1
W'_n: 1-ξ_1
W_n: ξ_4
W'_n: 1-ξ_4
W_n: ξ_2
W'_n: 1-ξ_2
W_n: ξ_3
W'_n: 1-ξ_3
ρ'
1-ρ'
ρ"
1-ρ"
< 1001 >
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Figure 1.

(A_1A_2) or (A_1A_3) is made available (with probability $\frac{1}{3}$) the subject will select A_1 , because the AAV is 1 for A_1 and 0 for both A_2 and A_3 . If a win follows the occurrence of the A_1 response (with probability ξ_1) no change in the AAV's occurs; however, if a loss terminates the trial (with probability $1-\xi_1$) then with probability ρ'' the AAV associated with A_1 becomes 0 and the new subject-state is $< 0001 >$. When the response pair (A_1A_4) is presented (with probability $\frac{1}{6}$) the subject selects the first response observed, since the AAV for both A_1 and A_4 are 1 . When response pair (A_2A_3) is presented both available responses have AAV's of 0 , and by Axiom 4 the subject randomly selects either A_2 or A_3 . The other paths of the tree are obtained in similar fashion.

Each path on the tree from a beginning point to a terminal point represents a possible outcome on a given trial. The probability of each path is obtained by multiplication of conditional probabilities. Thus in Figure 1 two paths lead from $< 1001 >$ to $< 0001 >$ and the corresponding transition probability is $\frac{1}{3}(1-\xi_1)\rho'' + \frac{1}{6}\frac{1}{2}(1-\xi_1)\rho''$.

Construction of the other trees yields a transition matrix for a sixteen state Markov chain. Certain states in the chain will be transient if some of the probabilities ξ_i are 0 or 1 .*/ However, in the experiment to be discussed this condition did not hold; therefore, our

*/ A state of a Markov chain is transient if the probability of ever returning to it is less than 1 .

comments will be confined to the case where the ξ_i are different from 0 and 1 . For this case, state $< 1111>$ is transient (in fact, the probability of re-entering it is 0) but the other 15 states form an irreducible, aperiodic chain. Thus, the limiting quantities $u_{ijk\ell}$ (i.e., the asymptotic probability of being in state $< ijk\ell >$) exist and are independent of initial conditions. Further when $\rho' = \rho'' = \rho$ it can easily be shown that $u_{ijk\ell}$ is also independent of ρ ; that is, $u_{ijk\ell}$ depends only upon the values of ξ_i set by the experimenter.

The data obtained in the experiment (see Suppes and Atkinson [1960, Ch. 11] for detailed information) indicated that the observed probability of an $A_{i,n}$ response approached a fairly stable value over the last 100 trials of the experiment.*/ Also the observed probabilities of an $A_{i,n}$ response given $D_n^{(ij)}$ approached stable values over the last 100 trials of the experiment. The corresponding theoretical predictions for choice behavior can be readily obtained. For example,

$$\lim_{n \to \infty} P(A_{1,n} | D_n^{(12)}) = u_{1011} + u_{1010} + u_{1001} + u_{1000}$$

$$+ \frac{1}{2}[u_{1110} + u_{1101} + u_{1100} + u_{0011} + u_{0010} + u_{0001} + u_{0000}] .$$

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*/ On early trials of the experiment the observed values of $P(A_{i,n})$ were approximately $\frac{1}{4}$ which would be expected if the subject initially had no preference among the four responses. The rate at which $P(A_{i,n})$ departs from its initial value and approaches an asymptotic level is of course determined by the reinforcement parameters ρ' and ρ'' .

And

$$P(A_{i,n}) = \sum_j P(A_{i,n}|D_n^{(ij)})P(D_n^{(ij)}) \quad .$$

As noted above, $u_{ijk\ell}$ is a function only of ξ_i when $\rho' = \rho''$. Consequently, in this special case, predictions for $\lim_{n\to\infty} P(A_{i,n}|D_n^{(ij)}) = P(A_i|D^{(ij)})$ and $\lim_{n\to\infty} P(A_{i,n}) = P(A_i)$ are entirely <u>a priori</u> and do not make use of any parameters evaluated from the data.

Table 1 presents the observed response proportions over the last block of 180 trials and the predicted asymptotic values for $\rho' = \rho''$. Overall the model gives a satisfactory account of the mean asymptotic response probabilities when predictions are based solely on experimentally determined parameter values. The correspondence between theory and data could be improved of course if ρ' and ρ'' were estimated from the data and used in generating predictions.

The agreement between these observed and predicted asymptotic response probabilities provides sufficient justification for the type of model construction considered in this paper. However, the model provides a much richer analysis of the experiment than the above results indicate. From the model we can predict not only average performance but also sequential properties of the individual subject's response protocol; i.e., the trial to trial increments and decrements in response probabilities.

Table 1
Predicted Asymptotic Values
and Observed Proportions
Over the Last Block of 180 Trials

	Predicted	Observed
$P(A_1 \mid D^{(12)})$	.442	.457
$P(A_1 \mid D^{(13)})$	.355	.445
$P(A_1 \mid D^{(14)})$	.228	.270
$P(A_2 \mid D^{(23)})$	.414	.375
$P(A_2 \mid D^{(24)})$	.286	.273
$P(A_3 \mid D^{(34)})$	.372	.368
$P(A_1)$	.171	.195
$P(A_2)$	.210	.199
$P(A_3)$	.267	.258
$P(A_4)$	.352	.348

It should be emphasized that one of the major contributions of mathematical learning theory has been to provide a framework within which the sequential aspects of learning can be scrutinized. Prior to the development of mathematical models relatively little attention was paid to trial by trial phenomena; at the present time, for many experimental problems such phenomena are viewed as the most basic aspects of learning data.

To indicate the type of sequential predictions that can be obtained from the model consider the probability of an A_1 response on trial $n+1$ given $D^{(12)}$ on both trial $n+1$ and trial n, a win on trial n, and an A_1 on trial n; namely $P(A_{1,n+1}|D_{n+1}^{(12)}W_nA_{1,n}D_n^{(12)})$. To obtain this result we proceed as follows:

$$P(A_{1,n+1}D_{n+1}^{(12)}W_nA_{1,n}D_n^{(12)}) = \sum_{j,k} P(A_{1,n+1}D_{n+1}^{(12)}C_{j,n+1}W_nA_{1,n}D_n^{(12)}C_{k,n})$$

where $C_{i,n}$ $(i = 1,\ldots,16)$ denotes subject-state i on trial n. In terms of our axioms we may rewrite the sum as

$$\sum_{j,k} P(A_{1,n+1}|D_{n+1}^{(12)}C_{j,n+1})P(D_{n+1}^{(12)})P(C_{j,n+1}|W_nA_{1,n}D_n^{(12)}C_{k,n})P(W_n|A_{1,n})$$
$$\cdot P(A_{1,n}|D_n^{(12)}C_{k,n})P(D_n^{(12)})P(C_{k,n})$$

$$= \frac{1}{36}\xi_1 \sum_{j,k} P(A_{1,n+1}|D_n^{(12)}C_{j,n+1})P(C_{j,n+1}|W_nA_{1,n}D_n^{(12)}C_{k,n})P(A_{1,n}|D_n^{(12)}C_{k,n})P(C_{k,n}).$$

Each of these quantities in the summation can readily be computed in terms of the axioms. For example, as n becomes large

$$P(A_{1,n+1}D_{n+1}^{(12)}W_nA_{1,n}D_n^{(12)}) \xrightarrow{n}$$

$$\frac{1}{36}\xi_1\left\{u_{1011}+u_{1010}+u_{1001}+u_{1000}+\frac{1}{4}[u_{1110}+u_{1101}+u_{1100}]\right.$$

$$\left.+\left[\frac{1}{2}\rho'+\frac{1}{4}(1-\rho')\right][u_{0010}+u_{0001}+u_{0000}]\right\} .$$

To obtain the appropriate conditional probability we divide this result by $\frac{1}{36}\xi_1 P(A_1|D^{(12)})$.

In terms of these sequential predictions various procedures can be devised for estimating the reinforcement parameters ρ' and ρ'' . Once these parameters have been estimated any theoretical quantity of interest can be computed and goodness-of-fit evaluations made. A consideration of these topics is not appropriate in this paper and the interested reader is referred to Suppes and Atkinson [1960].

At this point it would be nice if we could refer to a list of criteria and a decision rule which would evaluate the model and tell us whether this specific development or similar mathematical models are of any genuine value in analyzing the phenomena of interest to psychologists. Of course, such decision procedures do not exist. Only the perspective gained by refinement and extension of these models with empirical verification at critical stages will permit us to make such an evaluation. Certainly within the last decade almost all learning phenomena have been examined with reference to one or more mathematical models and there is no doubt that these analyses have led to a deeper understanding of the empirical findings. In addition, many new lines of experimentation have resulted directly from the work on mathematical models of learning. In spite of these developments some behavioral scientists maintain that psychology has not yet reached a stage where mathematical analysis is appropriate; still others argue that the data of psychology are basically different from those of the natural sciences and defy any type of rigorous systematization. Of course, there is no definitive answer to these critics. Similar objections were raised to mathematical physics as recently as the late 19th century, and only the brilliant success of the approach silenced opposition. A convincing argument is yet to be made for the possibility that mathematical models in psychology will not enjoy similar success.

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