PERFORMANCE MODELS OF AMERICAN INDIAN STUDENTS ON COMPUTER-ASSISTED INSTRUCTION IN ELEMENTARY MATHEMATICS

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P. Suppes, J. D. Fletcher, and M. Zanotti

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PERFORMANCE MODELS OF AMERICAN INDIAN STUDENTS ON COMPUTER-ASSISTED INSTRUCTION IN ELEMENTARY MATHEMATICS[1]

P. Suppes, J. D. Fletcher, and M. Zanotti[2]

Stanford University

This investigation applied experimentally the use of predictive-control models integrated into computer-assisted instruction (CAI) as discussed earlier by Suppes, Fletcher, and Zanotti (1973). Many of those who are engaged in curriculum reform efforts have been dissatisfied with classical evaluations that simply compare the pre- and post-treatment achievement of experimental and control groups. It is natural to seek a more predictive-control approach that can be used as an integral part of the curriculum in order to ensure greater benefits, especially for students who are educationally disadvantaged or handicapped and for whom global performance models derived from standard populations are inappropriate.

Such an approach was discussed by Suppes, Fletcher, and Zanotti (1973), who developed a theory by which the amount of time a student spends on a curriculum is a function of his progress, and his achievements in given course objectives, which are individually set for each student, are expressed as posttreatment grade placement (GP). Using the approach described by Suppes et al. (1973), we were able to achieve precise individualization of instruction both in the amount of instruction for each student and in the goal set for him. Further, the approach separates global features of the curriculum described by a simple differential equation from parameters that are characteristic of individual students.

THE IMSSS CAI SYSTEM

The central processor for the computer system for the Institute for Mathematical Studies in the Social Sciences (IMSSS) is a Digital Equipment Corporation PDP-10. In addition to 256K of core memory, short-term storage of programs and of student information is provided by sixteen 180,000,000-bit disk modules. Long-term storage of student response data is provided by magnetic tape. About 280,000,000 bits of information can be stored by the system on one magnetic tape. Communication with remote student terminals in participating schools is provided by private telephone lines. For communication with clusters of 16 or more terminals, high-speed data transmission and time-division multiplexing are used. About 110 CAI terminals can be used simultaneously with no appreciable detriment in the system's speed of response. Any curriculum or other program can be run at any time on any student terminal.

The student terminals are KSR Model-33 teletypewriters. These teletypewriters provide no audio, visual, or graphic capability, but their cost is about one-tenth of terminals that do. In a typical school, there is one room containing 8 to 15 student terminals. One person, the CAI terminal proctor, supervises both the use of the equipment and the students in the terminal room. Usually accompanied by their classroom teacher, the students enter and sit at any free terminal. Each student starts instruction by pressing a key to signal that he is positioned at the terminal and is ready to begin. The program responds by typing

HI

PLEASE TYPE YOUR NUMBER AND NAME.

and the student responds accordingly.

Each student receives a unique number when he enrolls for CAI, so the request for the first name is merely an additional safeguard to ensure correct identification. A student can be, and usually is, enrolled for several available CAI courses. He uses the same number for all courses and types a one-letter identifier to indicate which course he wants. The student in the following example types S, the identifier for the elementary-school mathematics strands course. Unless he types special instructions, the student begins exactly where he left off in the sequence of lessons. Student responses in the following example are underlined.

HI

PLEASE TYPE YOUR NUMBER AND NAME.

<u>S3456</u> <u>MARY</u> SMITH

JOB 10 ON TT5013 FRI FEB 2 73 8:46AM-PDT

HELLO MARY

HERE IS SESSION 46

16 - 9 = ____

THE ELEMENTARY-SCHOOL MATHEMATICS STRANDS CURRICULUM

The objectives of the curriculum are (a) to provide supplementary individualized instruction in elementary mathematics at a level of difficulty appropriate for each student's level of achievement, (b) to allow acceleration in any concept area in which a student demonstrates proficiency and to offer repeated drill in areas of deficiency, and (c) to provide a daily profile report of each student's progress through the curriculum.

A strand is a series of problems of the same operational type (e.g., number concepts, addition, subtraction, fractions) arranged sequentially in equivalence classes according to their relative difficulty. The 14 strands in the program and the grade levels spanned by each strand are presented in Table 1.

Insert Table 1 about here

A student in the strands program works on fewer than 14 strands; the actual number depends on his grade level and performance. The strands approach provides a high degree of individualization because each student's lesson is prepared for him daily by the computer, the lessons are presented as mixed drills at a level of difficulty in each strand determined by the student's prior performance, and the student moves up each strand at his own pace.

Details of the curriculum are documented by Suppes (1971), Suppes, Goldberg, Kanz, Searle, and Stauffer (1971), and by Suppes, Searle, and Lorton (1974).

METHOD

The approach used in this experiment was based on the Suppes, Fletcher, and Zanotti predictive-control models applied to CAI. Their theory of individual student progress was used to set individually appropriate GP goals for each student in the experiment, and then as a predictive mechanism, to regulate the amount of time spent on the curriculum by each student. All subjects in the experiment were scheduled by the school for one 10-minute mathematics strands session and for one 10-minute language arts

TABLE 1

Grade Level Spanned by Each Strand in the

Elementary Mathematics Program

Content	Grade level
Number Concepts	1.0-7.9
Horizontal Addition	1.0-3.9
Horizontal Subtraction	1.0-3.4
Vertical Addition	1.0-5.9
Vertical Subtraction	1.5-5.9
Equations	1.5-7.9
Measurement	1.5-7.9
Horizontal Multiplication	2.5-5.4
Laws of Arithmetic	3.0-7.9
Vertical Multiplication	3.5-7.9
Division	3.5-7.9
Fractions	3.5-7.9
Decimals	4.0-7.9
Negative Numbers	6.0-7.9

session each day. If a student was not progressing sufficiently toward his goal, he was automatically switched to the mathematics strands curriculum for an extra session when he requested his regularly scheduled language arts session; otherwise, he received the language arts session he requested. This implementation of the experiment required minimal intervention by the school administration or the CAI proctor.

Subjects

The subjects for this experiment were American Indian students living on a pueblo and attending a federally supported Bureau of Indian Affairs (BIA) school. Students chosen to participate in the experiment comprised all those who were taking at least one daily 10-minute session of the IMSSS language arts curriculum and at least one daily 10-minute session of the mathematics strands curriculum. Because a third-grade reading vocabulary was used in writing the language arts course (Fletcher & Beard, 1973), all students selected for the experiment were in the fourth, fifth, or sixth grades.

It is difficult to characterize a population of American Indian students other than to note that they are generally ill-equipped to cope with the values and content of middle-class Anglo education. Many of these students speak an Indian language in their homes, and many have adopted the rich culture of their tribe long before entering a BIA school. They typically score below grade level on most standardized tests of academic achievement.

Procedure

The experiment ran for 60 school days (February through May) in the 1972-73 school year and was administered as a routine part of the student's daily CAI sessions. Each student's performance history was examined daily to determine if he needed one or two mathematics strands sessions on the next day in order to reach the GP goal set for him. If he needed only one strands session on the next day, he received whatever curriculum he requested at sign-on; however, if he needed two sessions, he automatically received a strands session when he attempted to sign on to language arts. In this way, the administrative details of the experiment were controlled automatically by computer programs.

Two performance goals were set for each student. One goal was 'externally' derived, and one goal was 'internally' derived. Because the experimental period spanned about one-third of a school year, the external goal for each student was defined as a gain of .33 in GP. The predictive-control features of the experiment did not apply to the external GP goal. It should be noted, however, that a GP gain of .33 in 60 school days is an overly optimistic projection for students from this population.

The internal GP goal was more individualized than the external GP goal in that it was uniquely determined from each student's performance history, and the predictive-control aspects of this experiment were applied to these goals. In setting the internal performance goals, we examined the twentieth to fortieth strands sessions for each student to determine his average GP change per session. Average GP change per session for any student who had not received 40 strands sessions was determined by examining the twentieth to the latest CAI sessions. The internal GP goal for each student was then determined by extrapolation from these initial observations and from a linear model of his progress. Number of sessions taken and the internal GP goals were then used to integrate predictive-control techniques within the mathematics strands curriculum.

More specifically, the following model, adopted from the Suppes, Fletcher, and Zanotti discussion, was calculated for each student from observations of the twentieth to the fortieth sessions:

$$GP_i = a_i + b_i S_i^c \qquad c = \frac{1}{3} \tag{1}$$

where GP_i is average GP for student i,

S_i is number of sessions completed by student i.

Extrapolation was accomplished by assigning a value to S_i and solving for GP_i. Next, the probability that student i would take an assigned session was initially estimated from the twentieth to fortieth session observations:

$$P_i(S) = \frac{ED_{i,s_1-s_2}}{OD_{i,s_1-s_2}} \tag{2}$$

where $P_i(S)$ is the estimated probability that student i will receive an assigned session,

ED_{i,s_1-s_2} is the number of days required by student i to take session s_1 to session s_2 inclusive,

OD_{i,s_1-s_2} is the number of days required by student i to take sessions s_1 to s_2.

$P_i(S)$ was called a probability only for simplicity of conception. Clearly, if student i receives more sessions than are assigned to him, $P_i(S)$ would be greater than 1.0. Also, the variance of the $P_i(S)$ is greater than desirable; a student may miss two weeks because of illness and not miss another school day for the rest of the school year. For this reason the $P_i(S)$ were used only for assigning numbers of daily sessions to students and not for setting goal GP's.

The number of sessions, S_i, required by student i to reach his goal GP calculated using S_i from the model (1) of his performance on strands was then adjusted by $P_i(S)$.

$$AS_i = S_i + ((1 - P_i(S))*S_i) \qquad (3)$$

where AS_i is the number of sessions required by student i to reach his goal GP adjusted by the probability that he will receive an assigned session,

S_i is the number of sessions student i is expected to take in the experimental period,

$P_i(S)$ is the 'probability' student i will actually receive an assigned session.

Number of sessions required daily of student i is defined as

$$NS_i = \frac{AS_i}{DL} \qquad (4)$$

where NS_i is number of sessions required daily for student i,

AS_i is adjusted total number of sessions required by student i,

DL is number of days left in the experiment.

Number of daily sessions, T_i, actually assigned to student i is defined as

$$T_i = \begin{cases} 1 & \text{if } 0 \leq NS_i \leq 1.0 \\ 2 & \text{otherwise} \end{cases} \quad . \tag{5}$$

The procedures indicated by (2), (3), (4), and (5) were used for assigning students to one or two daily sessions of mathematics strands. The $P_i(S)$ were adjusted to include observations made during the experimental period as well as observations from the twentieth to fortieth session interval.

The extent to which the predictive-control mechanisms of this experiment depended on the accuracy and validity of average GP measured by the mathematics strands curriculum is notable. Suppes, Fletcher, Zanotti, Lorton, and Searle (1973) demonstrated that average GP measured by the strands program consistently underestimates GP measured by paper and pencil administrations of the Stanford Achievement Test (SAT) Arithmetic Computation subtest. This effect was observed even when subjects amassed large numbers of CAI sessions under experimental conditions, and it was corroborated by data presented by Suppes, Fletcher, and Zanotti (1973). It was assumed, therefore, that any implicit goals measured by standard administration of the SAT Arithmetic Computation subtest would be met, if not exceeded, by the operational definition of GP measured by the strands program used in this experiment.

RESULTS AND DISCUSSION

Complete data were obtained for 69 of the 70 subjects who began the experiment. Table 2 presents some descriptive data concerning the experiment. As the table shows, the subject population was expected to

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Insert Table 2 about here
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improve from a mean GP of 4.13 to 4.63, and this goal was generally exceeded as shown by the mean posttreatment GP of 4.69. For that matter, 62 of the 69 subjects, or 90 percent, exceeded their external goals, and 42 subjects, or 61 percent, exceeded their internal goals despite the fact that the subjects took an average of only 48.31 sessions during the experimental period. In designing the experiment we assumed that a subject could take as many as 100 sessions, if necessary, in the 60-day experimental period; in fact, the number of sessions taken ranged from 35 to 55.

It should also be noted in Table 2 that the mean number of sessions taken by the subjects before the experiment began was 84.52. Evidently the subjects were well into the curriculum before beginning the experiment, and the GP gains are not due to early, unusually rapid progress in the curriculum.

Investigation of the Theory

Suppes, Fletcher, and Zanotti (1973) emphasized that their differential equation is characteristic of the particular course to which it is applied and that the three individual parameters in the final model of student progress are to be estimated for each student individually. In the final model, of the form

TABLE 2

Descriptive Data Derived from the Experiment

	Mean	SD	N
Pretreatment strands GP	4.13	.92	69
Posttreatment strands GP	4.69	1.00	69
Goal GP	4.63	.93	69
Internal goal GP gain	.50	.16	69
Start of experimental sessions	84.52	8.82	69
End of experimental sessions	132.83	12.09	69
Sessions taken	48.31	4.77	69

$$GP_i(t) = a_i + b_i t^{k_i} ,$$

where t is the number of sessions and i is the student, the exponent k_i is the most important of the parameters to be estimated. We begin with estimates of a single constant k for the entire group of subjects. Table 3

Insert Table 3 about here
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shows how the mean standard errors of estimate (SEE), the mean absolute residuals, and the mean maximum residuals vary as k varies from .05 to 1.35 at intervals of .05. It is interesting to note, first, the curve of change in mean SEE; it ranges from .051 to .094, which represents only .043 of a change in GP. Second, the minimum for both the mean absolute residuals and the mean maximum absolute residuals occurs for about the same values of k, indicating that a 'minimax' strategy that seeks to minimize the maximum error in the predictive-control mechanism is probably compatible for this curriculum with a strategy that seeks to minimize the mean SEE over all students.

How much individualization provides significant returns is always an issue in selecting curriculum strategies. It is reasonable to ask if the values of k must be determined for each student or if sufficient individualization can be accomplished by assigning to every student a value for k that is simply the mean of all the values selected for k that are 'best' in that they are the values of k for each student that are associated with his minimum SEE over some range of fixed k. In this subject population, the mean of the 'best' k values is .637. Table 4

Insert Table 4 about here

TABLE 3

Mean Standard Errors, Mean Absolute Residuals, and

Mean Maximum Residuals for Different Values

of the Exponent k for the Group

k	Mean SEE	Mean absolute residuals	Mean maximum absolute residuals
.05	.086	.069	.154
.10	.081	.065	.146
.15	.076	.061	.138
.20	.072	.057	.131
.25	.068	.054	.124
.30	.064	.051	.118
.35	.061	.048	.112
.40	.058	.045	.107
.45	.055	.043	.103
.50	.054	.042	.100
.55	.052	.041	.098
.60	.051	.040	.098
.65	.051	.040	.098
.70	.052	.040	.100
.75	.053	.041	.102
.80	.054	.042	.106
.85	.056	.044	.111
.90	.059	.046	.117
.95	.062	.048	.123
1.00	.065	.051	.129
1.05	.069	.054	.137
1.10	.073	.057	.144
1.15	.077	.060	.153
1.20	.081	.064	.161
1.25	.085	.067	.169
1.30	.090	.071	.178
1.35	.094	.075	.186

TABLE 4

Comparative Data on Individual and Group Estimates of the Exponent k

	Mean SEE	Mean absolute residuals	Mean maximum absolute residuals
k = .637 for all students	.051	.040	.098
Individually best k for each student	.039	.030	.074

allows a comparison of the mean SEE, mean absolute residual, and mean maximum absolute residual when all k values are set to .637 with k values being estimated for each student individually. It is clear that individually estimating k yields some improvement in all three statistics.

Finally, the distribution of individually 'best' k values is of interest. A histogram for these values in the range .05 to 1.25 at increments of .05 is shown as Figure 1. Also included in the range of k

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Insert Figure 1 about here
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values is the model of the form

$$GP_i(t) = a_i + b_i \ln (t) ,$$

which Suppes et al. (1973) showed in one sense to be a lower limiting case for values of k. It is particularly interesting to note in Figure 1 the six instances in which $k > 1.00$. Despite the general, very slow negative acceleration of student progress in the curriculum, it is evident from these six cases that the progress of some students is positively accelerating even after more than 100 mathematics strands sessions. The total number of sessions taken by these six students ranges from 111 to 154.

SUMMARY AND CONCLUSIONS

From the analyses of data given above, we conclude that the mathematics strands CAI curriculum can lead to substantial increases in mathematics grade placement (GP) when used by American Indian students. The increases in grade placement exhibited during the 60 school days of

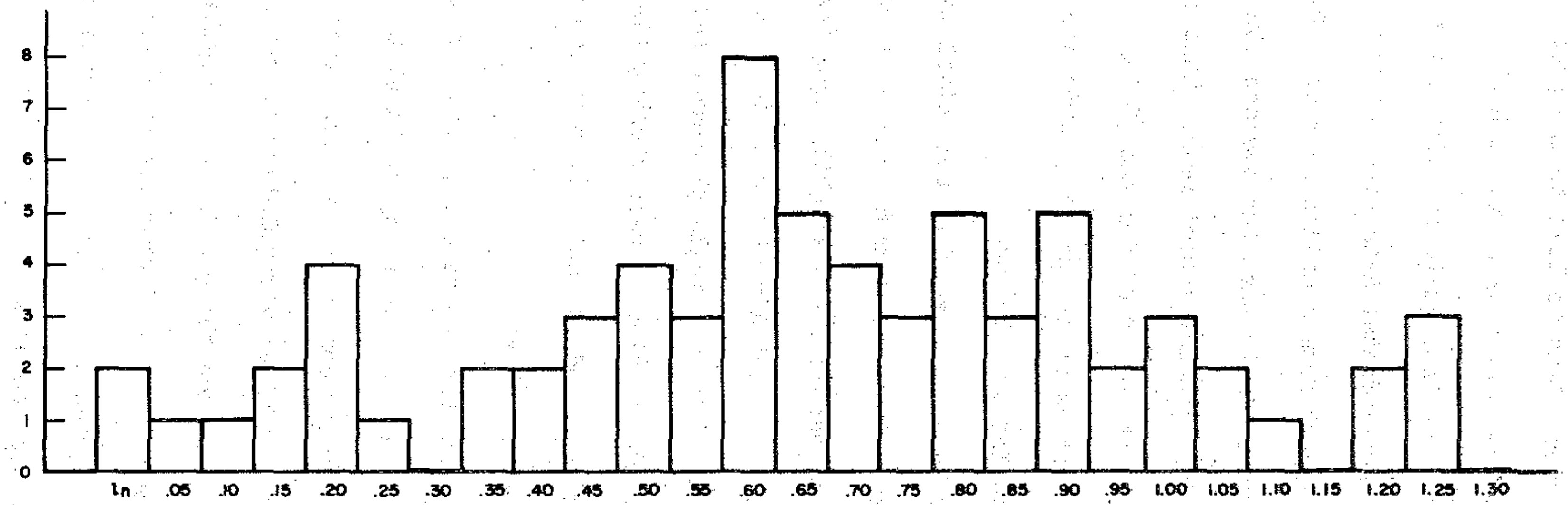

Fig. 1. Histogram for distribution of k in Isleta population.

the experiment were sufficient to lead to the prediction of more than one year's grade-placement gain in a full school year. Although the basic model that fits the data quite exactly is nonlinear, it is to be emphasized that the experimental data were not taken from the initial portion of the curve, but from a middle segment. It is a relatively good extrapolation to estimate that the average student gain in GP would be about 1.5 for a full 180-day school year. This estimate is based upon the average gain of .50 in GP for the 60 days of the experiment. It should be noted that gains of this magnitude are not ordinarily achieved by American Indian students.

It should also be emphasized that the actual time spent at a computer terminal by each student ranged from 6 to 10 minutes for each session, and the average number of sessions was slightly less than on a daily basis.

Finally, it is to be noted that the grade-placement gains reported in this experiment were for the grade-placement estimates in the strands curriculum, but the correlation between these grade placements and grade placements achieved on standard tests has now been examined intensively in earlier studies cited above.

It would seem important to determine whether the excellent grade-placement gains achieved in this study could also be achieved for longer periods of time and under conditions of less experimental control and greater variability. If the extrapolation of gains to longer periods of time is successful, then the provision of regular drill-and-practice CAI curriculum in mathematics affords one efficient method of significantly improving the grade-placement achievements of American Indian students in reservation schools.

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