PSYCHOMETRIKA—VOL. 24, NO. 2
JUNE, 1959

A MODEL FOR ORDERED METRIC SCALING BY COMPARISON OF INTERVALS*

ROBERT F. FAGOT†

UNIVERSITY OF OREGON

This paper presents a model of individual choice behavior for application to experimental situations in which a subject is required to compare utility intervals (differences in subjective value). This model is contrasted with a weaker model, which is also derived. Both models generate ordered metric scales, but differ in predictive power. An experiment on the utility of grades, which provides a test and comparison of the models, is presented.

Coombs [2] introduced the term ordered metric to denote those scales which provide an ordering of the set of alternatives, or stimuli (the single property of an ordinal scale), and in addition at least a partial ordering of the distances between stimuli (utility intervals). Siegel [8] presented a method, based on a one-person game, which generated a so-called higher-ordered metric scale, i.e., a complete ordering on utility intervals. The present model also generates a higher-ordered metric scale. The term ordered metric scale, as used in this paper, will always refer to a higher-ordered metric scale, as the term is used by Siegel [8], and will be abbreviated OM.

Coombs presented no formal characterization of the OM scale, nor did Siegel; one of the purposes of this paper is to present such a characterization. Two important results will be presented in relation to the properties of such scales. First, it will be shown that it is possible to construct a scale which satisfies the above two properties (an ordering of the set of stimuli, and an ordering of utility intervals), but which is inadmissible as an OM scale. This implies, of course, that the intuitive definition of an OM scale given above is inadequate, since it admits to the class of OM scales types which were not originally intended. Second, it will be shown that it is possible to have two or more models, all of which generate OM scales, but which differ in predictive power.

The subject will be required to make judgments in relation to pairs, triads, and tetrads of stimuli. For pairs of stimuli, he will have the traditional task of choosing one member of the pair on some specified basis. For triads

*This research was supported in part by Group Psychology Branch, Office of Naval Research, under Contract Nonr 225(17), with Stanford University, and in part by U. S. Public Health Service grant M-2046.

†The author wishes to express his indebtedness to E. W. Adams, University of California, whose constructive criticism did much to improve the quality of this work.

and tetrads of stimuli, he will be required to make comparative judgments of intervals. As an illustration, if three stimuli have been ordered by the method of pairs xyz, then the subject could be required to make the judgment whether y is more similar to x than to z. The particular relation utilized (in this illustration more similar) will depend on the experimental situation, but in all cases the subject will be required to make a judgment which permits an inference as to the ordering of two utility intervals. For example, if as above the stimuli are ordered xyz, and y is judged more like x than like z, then it will be inferred that y is closer on the subject's utility scale to x than to z; i.e., the utility interval between y and z is greater than the utility interval between x and y. If tetrads are used, the judgment will be of the form "the difference (along some specified dimension) between x and y is greater than the difference between z and w."

Although the fundamental relation between pairs of stimuli is interpreted in this paper in terms of direct comparisons of utility intervals, as illustrated above, the formal structure of the model is applicable to any method, such as Siegel's one-person game [8], for which each judgment implies an ordering of two utility intervals.

Formal Structure of the Model

The model is based on three primitive notions. The first is a set K, interpreted as the set of alternatives, or stimuli. The second is a quaternary relation Q on the cartesian product $K \times K$. The relation Q is interpreted such that $xyQzw$ holds when the difference in subjective value (utility) between x and y is judged *greater than* the difference between z and w. If the alternatives are objects among which the subject is stating preferences, then $xyQzw$ implies that the difference in strength of preference (value) between x and y is greater than the difference between z and w; the subject more strongly prefers x to y than z to w. If the four alternatives are tones which differ only in frequency, then $xyQzw$ implies that the difference in pitch between x and y is judged by the subject to be greater than the difference in pitch between z and w. The third primitive is a binary relation P on the set of alternatives K. P is the relation of *strict preference*; xPy holds if and only if the subject strictly prefers x to y.

A numerical interpretation of the relations P and Q is given with the following definition of a quaternary utility function.

Definition 1. Let U be a real-valued function defined over K; then U is a *quaternary utility function* if it satisfies the following conditions for every x, y, z, and w in K:

(a) xPy if and only if $U(x) > U(y)$;

(b) $xyQzw$ if and only if $U(x) - U(y) > U(z) - U(w)$.

The basic assumption of the model may now be stated.

Quaternary Hypothesis (QH). There exists a quaternary utility function for the relations P and Q and set K, with the following restrictions.

(a) For every x and y in K, if $x \neq y$, then $U(x) \neq U(y)$.

(b) For every x, y, z, and w in K, if $U(x) > U(y)$ and $U(z) > U(w)$ and $x \neq z$ or $y \neq w$, then $U(x) - U(y) \neq U(z) - U(w)$.

Restrictions (a) and (b) are necessary if the relations P and Q are to have the desired ordering properties, which will be clear after consideration of empirical consequences 1 and 3, presented in the following section. Condition (b), of course, rules out the possibility that an interval $U(x) - U(y)$ equals some other interval $U(z) - U(w)$. However, this exclusion is consistent with the intended application of the model to situations of strict preference.

For every choice that a subject makes, we may write a corresponding inequality in utility values (Definition 1). An experiment would produce a set of inequalities for each subject, and a quaternary utility function exists for a particular subject (the model holds exactly) if and only if the resulting set of inequalities is consistent, i.e., has a solution. There are well-known decision procedures for determining whether or not a set of inequalities has a solution [e.g., 6].

Satisfaction of the QH is sufficient to guarantee the existence of an OM scale, but it is not necessary. This follows from the fact that, as will be shown in the next section, it is possible to derive from the QH consequences which, if satisfied, guarantee the existence of an OM scale, yet these consequences do not exhaust the empirical significance of the QH. This means that the QH may be disconfirmed even if an OM scale exists. Hence one important problem is to find the weakest conditions imposed by the QH which are capable of generating the desired OM scale. An allied problem is to define progressively stronger alternative models in terms of the empirical consequences of the basic hypothesis, so that it will be possible to state precisely how strong a model is sustained by the data. In the following section some of the empirical consequences which follow from the basic hypothesis are specified and in the process two alternative models are defined.

Some Empirical Consequences of the Quaternary Hypothesis

By empirical or observable consequence of the QH is meant any consequence implied by the QH about the P and Q relations themselves. Unfortunately, a result obtained by Scott and Suppes ([7], pp. 16–27) implies that there is no simple set of consequences of the type to be considered which completely exhausts the empirical significance of the QH for all finite sets of stimuli. Derivation of the empirical consequences, and proofs of certain propositions, such as the independence of these consequences, will not be presented, since they are mostly simple, tedious, and mathematically uninteresting.

The first consequence of the QH is that the binary relation P is a *strict simple ordering* of K. The terminology follows that of Suppes [9].

Empirical Consequence 1 (C1). The binary relation P is a *strict simple ordering* of K; P satisfies the following conditions for every x, y, and z in K:

(a) exactly one of the following holds: $x = y$, xPy, yPx;
(b) if xPy and yPz then xPz.

Thus if C1 is satisfied it becomes possible to derive a ranking without ties of the alternatives in K.

One is perhaps used to thinking of this consequence as the sole axiom of an ordinal model, but certain Q relations are predictable from P relations, and hence the ordinal model must be defined to include an axiom formulating the connection between P and Q where it is possible to predict Q preferences from P preferences. For example, if three alternatives are ordered $xPyPz$, then this knowledge alone implies that the distance between x and z is greater than the distance between x and y, and also greater than the distance between y and z. This notion is formalized below.

Empirical Consequence 2 (C2). For every x, y, z, and w in K,

(a) if xPy and yPz, then $xzQxy$ and $xzQyz$;
(b) if xPy, yPz, and zPw, then $xwQyz$.

In most preference experiments, one would hardly expect C2 to be disconfirmed if C1 is satisfied, but this confidence might not be at all sustained in certain psychophysical experiments. In any event, C1–2 are the axioms of the *Ordinal Model*.

One further consequence of the basic hypothesis permits the formulation of a model which generates an OM scale. Consider a subset P of $K \times K$ such that for all $\langle x, y\rangle$ in $K \times K$, $\langle x, y\rangle$ belongs to P if and only if xPy. P then is the subset of $K \times K$ which is the set of ordered pairs xPy. Then one obvious consequence of the QH is that Q is a strict simple ordering of P.

Empirical Consequence 3 (C3). The quaternary relation Q is a *strict simple ordering* of P (Q is asymmetric, transitive, and connected in P); i.e., for every $\langle x, y\rangle$, $\langle z, w\rangle$, and $\langle u, v\rangle$ in P,

(a) exactly one of the following holds: $\langle x, y\rangle = \langle z, w\rangle$, $xyQzw$, or $zwQxy$;
(b) if $xyQzw$ and $zwQuv$, then $xyQuv$.

The intuitive statement made in the introduction that a complete ordering on utility intervals is required for an OM scale corresponds to the formal requirement that Q be a strict simple ordering of P. The restriction to P is a restriction to positive differences in utility.

The fact that C2 and C3 are independent has some interesting implications. It will be recalled that the intuitive definition of an OM scale requires only that C1 and C3 be satisfied. However, C2 could be disconfirmed even if C1 and C3 were satisfied. Hence C1 and C3 are not sufficient for the derivation of an OM scale; C2 is necessary. This is the point of the remark made in the introduction that the intuitive definition of an OM scale as requiring an ordering of the stimuli and an ordering of utility intervals was inadequate.

C1–3 do not exhaust the empirical significance of the QH. This fact, coupled with the fact that these consequences are sufficient to generate an OM scale, leads to the important result that an OM scale may exist even when the QH is disconfirmed. Such a model is called a *weak* ordered metric model to distinguish it from the stronger model which assumes the existence of a quaternary utility function. Empirical consequences 1, 2, and 3 define the *Weak Ordered Metric Model* (hereafter abbreviated as WOM); i.e., C1–3 are the axioms of the WOM. It should be noted that although the term weak is used, the model still generates a higher-ordered metric scale.

Although C1–3 are the set of necessary and sufficient conditions for the existence of an OM scale, it is of considerable interest to formulate further consequences of the QH, for a number of reasons. First, completely separate from the problem of measurement, these consequences could be used for the prediction of choice behavior. Second, if the QH is disconfirmed, then it becomes possible to state precisely in which way it was disconfirmed by examining each of the consequences. Third, the formulation of such consequences permits the construction of progressively stronger models of choice behavior which can be compared in experimental situations. Finally, the two consequences to be presented exhaust the empirical significance of the QH under certain special conditions to be discussed after their presentation. The first of these consequences is presented below.

Empirical Consequence 4 (C4). For all x, y, z, and w in K, if $xyQzw$ then $xzQyw$, $wyQzx$, and $wzQyx$.

This consequence is stated in its most general form, but in the usual experimental situation the observation $xyQzw$ implies xPy and zPw, in which case the consequence could be simplified to "if $xyQzw$ then $xzQyw$" for direct prediction purposes. In any event, for any given ordering of stimuli, C4 will have only one conclusion with empirical content, not three. Its empirical content corresponds to the arithmetical fact that $a > b$ if and only if $a + x > b + x$. For example, suppose four stimuli are ordered $xPyPzPw$. Corresponding to "if $xyQzw$, then $xzQyw$" is, writing capital letters for the utilities of the stimuli, "if $X - Y > Z - W$, then $X - Z > Y - W$." The conclusion can be written $(X - Y) + (Y - Z) > (Y - Z) + (Z - W)$, from which it can be seen that the interval $(Y - Z)$ is common to both sides of the inequality, which can be reduced to $X - Y > Z - W$, which is of course, the premise.

The following example shows that C4 has implications which C1–3 do not. Suppose again that four stimuli are ordered $xPyPzPw$, and further that the six utility intervals are ordered $xwQxzQywQzwQxyQyz$. P and Q are transitive and hence C1 and C3 are satisfied. Inspection of the ordering of the intervals indicates that C2 is also satisfied. Note, however, that the observation $xzQyw$ implies $xyQzw$ according to C4. But $zwQxy$ was observed; therefore C4 is disconfirmed. This means that C4 has implications which C1–3 do not. This result confirms the important conclusion already stated, that C1–3 do not exhaust the empirical significance of the QH, in spite of the fact that these consequences are sufficient to generate an OM scale.

One final consequence of the QH is now presented.

Empirical Consequence 5 (C5). For all x, y, z, w, u, and v in K, if $xyQzw$ and $uxQwv$, then $uyQzv$.

The empirical content of this consequence is readily seen if the relations are written in terms of the corresponding utility values: if $X - Y > Z - W$ and $U - X > W - V$, then $U - Y > Z - V$. Adding the two inequalities of the premise, $(U - X) + (X - Y) > (Z - W) + (W - V)$, which of course simplifies to $U - Y > Z - V$, the conclusion. The arithmetical interpretation is (i) if $m > n$ and $r > s$, then $m + r > n + s$. In a similar way one can show that C5 also has the arithmetical interpretation (ii) if $r > n$ and $m + n > r + s$, then $m > s$.

When an implication of C5 involves four stimuli (e.g., $abQcd$), then this Q relation can also be predicted by C4, although the two predictions may differ. Thus one of the subjects in the experiment to be reported made the following choices: $dfQab$, $bfQad$, $cdQbc$, and $acQcf$. The first choice implies the second by C4, but the third and fourth imply $adQbf$ by C5, which contradicts the second. This shows that C5 has implications which C1–4 do not, since the total set of observations was consistent with C1–4, but C5 was disconfirmed.

If a C5 implication involves only three stimuli (e.g., $xyQyz$), then it cannot be predicted by C4. It also follows that in such a case the premises of C5 must involve only *five* stimuli, not six. Therefore the number of predictions which can be made by C5 but *not* by C1–4 cannot exceed $\binom{n}{5}$, where n is the number of stimuli in K. However, even if a C5-implication is not predictable by C4, it may be predictable by C3. Since there are $\binom{n}{2}$ ordered utility intervals, there will be exactly $\binom{n}{2} - 1$ Q relations which are not predictable by C3. Therefore the *maximum* number of predictions which can be made by C5 but not by C1–4 is $\binom{n}{5}$ or $\binom{n}{2} - 1$, whichever is smaller. This upper bound reflects the maximum additional predictive power of a model which contains C5 as an axiom as well as C1–4.

Although C1–5 do not exhaust the empirical significance of the QH for all finite sets of stimuli, a weaker statement on sufficiency conditions can be made: if restriction is made to predictions based on not more than *two* choices (i.e., of the form "if A then B" or "if A and B then C"), then C1–5 do in fact exhaust the empirical significance of the QH. This means that under such a restriction, any prediction which can be made with the QH can also be made with C1–5. The increase in predictive power resulting from the utilization of more than two choices is slight for small sets of stimuli (less than ten), and hence this is an important result.

In formulating the WOM, the set of necessary and sufficient conditions for the existence of an OM scale have been specified. Such a model may be compared in experimental situations with the *Strong Ordered Metric Model* (hereafter abbreviated SOM), which assumes the existence of a quaternary utility function. Such a strong model is perfectly satisfied if and only if the set of inequalities in utility values corresponding to the set of observed P and Q relations has a solution. However, as pointed out, even if the solution does not exist, the WOM may be satisfied, and hence an OM scale may exist.

Relation to Two Other Theories

The present model will be discussed in relation to two other theories. The first is that used by Siegel [8] to obtain an OM scale, and the second is a model of riskless choice developed by Adams and Fagot [1].

The essential device underlying Siegel's approach is a one-person game in which the subject chooses between two options, each of which is a probability combination of two outcomes. A chance event E, which has an experimentally determined subjective probability of one-half, is defined. The subject is required to choose between two options (x, y) and (z, w). If he chooses the first and E occurs he gets x, while if E does not occur he gets y. The second option is determined in a similar manner.

If one now introduces a quaternary relation R such that $xyRzw$ holds if and only if the subject chooses the option (x, y) when presented with a choice between (x, y) and (z, w), then condition (b), Definition 1, of a quaternary utility function may be modified as follows:

(b′) $xyRzw$ if and only if $U'(x) + U'(y) > U'(z) + U'(w)$.

Condition (b′) is based on the assumption that an individual makes choices among alternatives involving risk as if he were trying to maximize expected utility ([8], pp. 212–213).

Thus there are two methods for deriving OM scales, and it would be of considerable interest to determine if the same scales would be derived by both methods. The following hypothesis postulates a relationship between R and Q.

RQ Hypothesis. For every x, y, z, and w in K, $xyRzw$ if and only if $xwQzy$.

This hypothesis is equivalent to the assumption that the utility functions U and U' are the same. Given a set of axioms on Q (such as C1–5), these axioms can be immediately transformed to a set of axioms on R, by means of the RQ hypothesis. It then becomes possible to determine if individual axioms hold for both Q and R, rather than simply testing the stronger RQ hypothesis. It should be noted that the existence of U' is not necessary for the derivation of an OM scale; satisfaction of axioms equivalent to C1–3 is sufficient.

There are several interesting possibilities which could arise from experimental comparisons of the two methods. For example, both U and U' may exist, yet not be the same. Or, neither U or U' may exist, yet an OM scale may exist for both (i.e., the WOM may be satisfied for both Q and R observations), and this scale may not be the same. Finally, one method may produce an OM scale, but not the other.

Another theory, formally quite similar to the present model, is a theory of riskless choice developed by Adams and Fagot [1]. This theory is concerned with subjects' choices between alternatives which are multidimensional, although the theory has been worked out only for the two-dimensional (or two-component) case. Such alternatives might be political candidates who are described as varying in two characteristics, liberality and foreign policy, for example. In this case the subject would be required to state a preference for one of every pair of such candidates. Or, in an extension, the alternatives may be pairs of objects of any sort. For example, one two-component alternative could be book x and book y, and the subject might have to choose between this alternative and a second which consisted of two books z and w. It is clear then that the components of the two-component alternatives can be thought of as belonging to a set K of alternatives, such as has been considered previously in this paper. The model assumes that the individual behaves as if he assigns subjective values (utilities) to each of the components *independently*, and then *adds* the values together to get the value of the composite alternative. The fundamental assumption of the model is the hypothesis of the existence of this *additive* utility function. Comparable to condition (b), Definition 1,

(b″) $(x, y)P(z, w)$ if and only if $U''(x) + U''(y) > U''(z) + U''(w)$, where P is the relation of strict preference.

The model generates an OM scale, and a simple transformation relates this model to the present model. This relationship is indicated by the following hypothesis.

PQ Hypothesis. For every x, y, z, and w in k, $(x, y)P(z, w)$ if and only if $xzQwy$.

This hypothesis is equivalent to the assumption that the utility functions U and U'' are the same. All three models are related by the following corollary.

COROLLARY OF RQ AND PQ HYPOTHESES. For every x, y, z, and w in K, $(x, y)P(z, w)$ if and only if $xyRzw$.

It should be realized that R holds between options and P holds between alternatives. Thus in the case of P the outcome is x *and* y, whereas in the case of R the outcome is x *or* y, depending upon the outcome of a chance event E.

In this additive model of riskless choice, the basic assumption of the additivity of the components is usually used simultaneously to determine the utility values and to make predictions. Under these conditions, the predictive consequences of the model are much weaker than those using utility values obtained outside the model by independent methods of measurement. Observations on Q or R relations could provide this independent method of measurement. In other words, U or U' could be used to assign utility values to the alternatives in a basic set K, and then the additive model could be used to make predictions between *pairs* of these alternatives, under the assumption that the utility values are additive.

Some Experimental Results

Some results of an experiment on the utility of course grades afford a test of the model and illustrate the derivation of an OM scale. Ten students in introductory psychology served as subjects. The alternatives in the set K are the course grades A, B, C, D, and F. Appropriate operational definitions were given to the relations P and Q such that aPb means that the subject prefers the grade A to the grade B, and $dfQab$ means that the difference, in value (utility), between the grades D and F is greater than the difference between the grades A and B. Care was taken to insure that each subject thoroughly understood the operation of comparing two utility intervals and reporting the interval which was subjectively greater.

Let n be the number of stimuli. Then N_0, the number of Q relations predictable from a knowledge of an ordering of the stimuli, is

$$N_0 = 2\binom{n}{3} + \binom{n}{4}. \tag{1}$$

In this experiment there are five stimuli in K; if all comparisons are made, the complete data for each subject would consist of 10 P relations and 45 Q relations. Of these 45 observations, 25 would be predictable from C2 ($N_0 = 25$). However, in a preference experiment of this sort, there seems little necessity for making a complete test of this consequence, so only 5 such comparisons were presented to each subject. The responses of each of the ten subjects were consistent with C2. C1 was not tested; it was assumed that all subjects preferred the grades in the order $aPbPcPdPf$.

The choices of one of the subjects will be selected for discussion. Table 1 lists the 20 choices of this subject which cannot be predicted from a knowledge of an ordering of the stimuli, and hence these observations contain the ordered metric information.

These choices are divided into subsets by means of C4; subset B is predictable from subset A by means of C4. Utilizing this consequence one can predict (16) from (2), (17) from (5), (18) from (3), (19) from (12) and (20) from (8); thus C4 is perfectly satisfied.

TABLE I

Observations of One Subject in Experiment on Utility of Grades

Subset A						Subset B	
1.	abQbc	6.	bfQab	11.	acQcd	16.	acQbd
2.	abQcd	7.	bcQcd	12.	dfQac	17.	bfQac
3.	dfQab	8.	dfQbc	13.	cfQac	18.	bfQad
4.	bdQab	9.	cfQbc	14.	adQdf	19.	cfQad
5.	cfQab	10.	dfQcd	15.	dfQbd	20.	cfQbd

A necessary and sufficient condition for admission of an observation to subset A is that the two intervals corresponding to the two ordered pairs of a Q relation do not overlap. For example, if $xPyPz$, then the only such relation is $xyQyz$ (or $yzQxy$, whichever is observed). The number of Q relations in subset A is

$$N_A = \binom{n+1}{4}. \qquad (2)$$

The two intervals for a Q relation in subset B always overlap. For example, observation (16) implies $A - C > B - D$, and it is clear that these two intervals have the interval $B - C$ in common. The number of observations in subset B, and hence the number of predictions that can be made in any experiment by means of C4, provided that certain observations are made in subset A, is

$$N_B = \binom{n}{4}. \qquad (3)$$

It is important to note that although N_B choices can be predicted from certain choices in subset A, these predictions cannot be made by the WOM, since C4 is not an axiom in this model.

Inspection of Table 1 shows that the choices of this subject were consistent with C3 (Q is transitive), and with C4. He satisfies not only the WOM but also the SOM (the set of inequalities corresponding to the observations have a solution). The OM scale which is derived from the observations in Table 1 is defined by the following ordering of utility intervals

$$(4) \qquad A - F > B - F > C - F > A - D > D - F > A - C > B - D > A - B > B - C > C - D.$$

In general, if there are i intervals, then $(i - 1)$ Q relations are sufficient to define the OM scale—but not always necessary. For example, of the nine Q relations corresponding to (4), the following seven are sufficient to generate the OM scale by the WOM: *cfQad*, *adQdf*, *dfQac*, *acQbd*, *bdQab*, *abQbc*, *bcQcd*. The relations *afQbf* and *bfQcf* are not necessary, since they are implied by the P relations by means of C2. Thus the remaining 38 Q relations contain superfluous information since they are predictable from these seven by means of the axioms of the WOM, thus providing a test of the model. Depending on the structure of the scale, as few as three observations may in some cases provide sufficient information for the construction of the OM scale. Unfortunately, precisely which observations provide this information cannot be specified in advance. However, Siegel ([4], p. 141) has developed a "maximin" method which maximizes the number of predictions which may be made from observing a minimum number of choices, and specifies the order in which choices should be examined in constructing the scale. His method may be adapted for use with the present model.

The number of observations necessary and sufficient to represent the entire set of observations can be taken as a measure of the *predictive power* of the model. The smaller the number of such observations, the greater is the predictive power of the model. In the case above, two of the observations, *cfQad* and *acQbd* are not necessary in the SOM since they are implied by *dfQac* and *abQcd*, respectively, by C4. Thus in the SOM only five observations are necessary, whereas in the WOM seven are necessary, indicating the correspondingly greater predictive power of the former. In this experiment the number of choices necessary and sufficient to represent the total set of 45 observations ranged, in the WOM, from a low of six for one subject to a high of eight for another subject, and in the SOM, from a low of three to a high of five, the lower values of the SOM being indicative of its greater predictive power. This concept of predictive power is treated more extensively in other sources ([5]; [3], pp. 28–36).

The general results of the experiment are as follows.

1. Seven of the ten subjects perfectly satisfied the WOM; hence for these seven subjects it was possible to construct an OM scale.

2. Two of these seven failed to satisfy the SOM. These two subjects are of special interest, since for them it was possible to construct an OM scale in spite of the fact that they did not satisfy the QH. One of these two had a

single disconfirmation of C4, and the second had a single disconfirmation of C5.

3. Seven of the ten subjects perfectly satisfied C4 (five times each); the remaining three each had a single disconfirmation.

4. At most *one* prediction for each subject can be made by C5 which cannot be made by means of C1–4, since there are only five stimuli in this experiment. The results were that one such prediction was made for each of three subjects, and one of these predictions was disconfirmed. (If the number of stimuli were increased by one to six, then the maximum number of implications of C5 for each subject would increase from one to six.)

5. For each of the three subjects who did not perfectly satisfy the WOM, the reversal of a single choice would have resulted in satisfaction of the model; however, one of these three still would not have satisfied the SOM. Methods of analyzing the fit of the model have been devised, but discussion will be deferred to a later paper on the analysis of a series of such experiments.

REFERENCES

[1] Adams, E. W. and Fagot, R. F. A model of riskless choice. *Behav. Sci.*, 1959, 4, 1–10.
[2] Coombs, C. H. Psychological scaling without a unit of measurement. *Psychol. Rev.*, 1950, **57**, 145–158.
[3] Fagot, R. F. An ordered metric model of individual choice behavior. Tech. Rep. No. 13, Contract Nonr 225(17), Stanford Univ., 1957.
[4] Hurst, P. M. and Siegel, S. Prediction of decisions from a higher ordered metric scale of utility. *J. exp. Psychol.*, 1956, **52**, 138–144.
[5] Kemeny, J. and Oppenheim, P. Systematic power. *Phil. Sci.*, 1955, **22**, 27–33.
[6] Kuhn, H. W. Solvability and consistency for systems of linear equations and inequalities. *Amer. math. Mon.*, 1956, **63**, 217–232.
[7] Scott, D. and Suppes, P. Foundational aspects of theories of measurement. Tech. Rep. No. 6, Contract Nonr 225(17), Stanford Univ., 1957.
[8] Siegel, S. A method for obtaining an ordered metric scale. *Psychometrika*, 1956, **21**, 207–216.
[9] Suppes, P. *Introduction to logic.* Princeton: Van Nostrand, 1957.

Manuscript received 9/29/58

Revised manuscript received 10/24/58