OPEN PROBLEMS IN RELATIONAL GRAMMAR

ABSTRACT. The notion of a relational grammar was proposed by Suppes for the semantics of natural language. A relational grammar is a context-free grammar with a relation algebraic semantics. While this concept provides elegant solutions for the semantic analysis of a variety of constructions, certain problems arise with its application to certain other constructions. In this paper, the focus is on proper nouns, pronouns, number, and verb phrases.

The notion of relational grammar (henceforth: RG) was proposed in Suppes (1976) for the semantics of natural language and has been expanded on in subsequent articles. RG was introduced for the purpose of giving a syntactic and semantic analysis of natural language.

The most attractive feature of RG is its closeness to the structure of natural language. The meaning of an expression can be read off from its linguistic structure: the sentence

\[
\text{All men like some women}
\]

gets the semantic structure

\[
f([\text{men}], [\text{like}], [\text{women}])
\]

where \([\alpha]\) stands for the denotation of \(\alpha\) and \(f\) is a semantic function. No use is made of variables for individuals as in quantifier logic. RG is first order, i.e. \([\alpha]\) is either a subset of some domain \(D\) or a binary relation on \(D\). It thus avoids the procedural complexities involved in the computation of sets higher up in the set-theoretical hierarchy. The operations employed in RG semantics are the familiar set-theoretical operations with well-known properties that enhance the transparency and expedition of logical inferences. RG does not require any level of semantic representation, but denotations are mounted directly to natural language structures.

For all these reasons I believe that G is an interesting and attractive alternative to current theories of linguistic semantics. The purpose of
this contribution is to point out some problems and difficulties arising from the application of RG to natural language. The organization of this paper is as follows: in Section 1 I shall give a short introduction to RG. In Section 2 the problem of the missing category NP is addressed. In Section 3 the problem of anaphoric pronouns is taken up. In Section 4 the problem of verbal semantics is addressed.

1. RELATIONAL GRAMMAR

An RG is a context free grammar $G$ with semantic functions $f$ attached to each production rule

$$A \rightarrow A_1 + \cdots + A_n \quad [A] = f([A_{i1}], \ldots, [A_{ik}])$$

where $\{i_1, \ldots, i_k\} \subseteq \{1, \ldots, n\}$ and denotations are restricted to elements of an extended relation algebra over the domain $D$ as some model structure for $G$. The purpose of the functions $f$ is to provide a compositional semantics for $G$. For details see Suppes (1973a).

An extended relation algebra over $D$ is any collection of subsets of $D$ and binary relations on $D$ that is closed with respect to the operations of

1. union: $X \cup Y$
2. complementation: $-X$
3. conversion of a binary relation: $\check{R}$
4. composition of binary relations $R$ and $S$: $R; S$
5. (upper) image of set $A$ under relation $R$: $R^{\uparrow}A$

where $X, Y$ stand for either subsets of $D$ or binary relations on $D$. An extended relation algebra is thus a subset of the following set-theoretical hierarchy

$$(1) \quad \mathcal{P}(D) \cup \mathcal{P}(D \times D)$$

where $\mathcal{P}(D)$ denotes the power set of $D$. An equational characterization of extended relation algebras can be found in the notion of a Boolean module (Brink, 1981) which is a Boolean algebra together with a set of operators forming a relation algebra in the sense of Tarski (1941).

The operations (1) give rise to numerous other operations that can be defined in terms of these. It is well known that we can define
Boolean product, Boolean difference, and symmetric difference by Boolean union and complementation. Less known may be the following operations:

\[
\begin{align*}
  f_1(R, A) &= -((-R)^c A) \\
  f_2(R, A) &= -(-\bar{R}^c A) \\
  f_3(R, A) &= -(\bar{R}^c - A) \\
  f_4(R, A) &= -(R^c - A) \\
  f_5(R, A) &= A \cap \bar{R}^c \{c\}.
\end{align*}
\]

$f_1$ was introduced in Riguet (1948) under the term *Coupe de deuxième espèce de R suivant A*; $f_2$ was known already to de Morgan and Peirce under the term of *Ordinary Involution*, as pointed out in Brink (1978); $f_3$ was also known to de Morgan and Peirce under the term *Backward Involution*, cf. Brink (1978); $f_4$ was introduced by Suppes and Zanotti (1977) as the *Lower Image RccA* of $A$ under $R$; $f_5$ was introduced in Suppes and Macken (1978) to compute the *initial segment IS (A, R, c)* of a set $A$ with respect to an ordering relation $R$ on $A$ and an element $c$ of $A$.

Notice that there are set-theoretical operations that cannot be defined in terms of (1) like, for example, domain restriction, Cartesian product, or cylindrification. One might suppose that they do not play an important role at all in the semantics of natural language. For the operation of Cartesian product, this was indeed pointed out in Keenan (1983, p. 71). On the other hand, there may be some use for cylindrification, cf. Woods (1991), and for restriction in a procedural semantics, cf. Böttner (1992b).

RG was meant to serve as a tool for the analysis of everyday natural language. Since RG provides only two kinds of denotations, namely sets and binary relations, any well-formed natural language expression denotes either a set or a binary relation, provided it has a denotation at all. Sets are denoted by common nouns, proper nouns, classifying adjectives, intransitive verbs, verb phrases, and prepositional phrases. Binary relations are denoted by transitive nouns, intensifying adjectives, and prepositions. Certain words, like determiners and conjunctions, do not denote at all. They are treated syncategorematically.
For the purpose of illustration let me give an example that to my knowledge has not been used before. In Woods (1991) a certain operation called MR is introduced to derive a new concept

(3) \( \text{MR[child][doctor]} \)

from a relational term like, e.g. \textit{child}, and an absolute term like, e.g. \textit{doctor}. The meaning of the derived concept is given in natural English by

(4) \textit{all of whose children are doctors}.

In terms of conventional grammar (4) is a relative clause that can modify a noun. One of the characteristic features of this relative clause is that the relative pronoun is in the possessive case. It turns out that MR corresponds exactly to the lower image operation of Suppes and Zanotti (1977). Although no precise grammar for natural language is given by Woods, I think that such a grammar can and indeed should be given and that RG is most appropriate for this purpose. An RG that derives (4) would be:

(5) \[
RC \rightarrow \text{all of whose } RN + VP \\
[RC] = [RN] \cdot [VP] \\
VP \rightarrow \text{are } N \\
[VP] = [N]
\]

where \( RC \) = relative clause, \( RN \) = relational noun and \textit{children} is a relational noun. To derive the denotation of the phrase \textit{student all of whose children are doctors} is completely standard using intersection.

2. PROPER NOUNS

One of the consequences of the radical restriction of the hierarchy of denotations is that RG cannot have combinations of a determiner and a common noun phrase as a constituent. A rigorous proof for that has been given in Suppes (1976). This is in striking contrast to conventional linguistic analysis where combinations of determiner and common noun are grouped as one constituent and classified under the category \( NP \) (noun phrase). In contradistinction, proper nouns do not combine with a determiner but are also classified under category \( NP \). It is considered to be one of the merits of Montague grammar to provide a
semantic category corresponding to the syntactician's category NP and allowing the treatment of proper nouns on a par with determiner noun combinations.

Whereas the coordination of predicates or verb phrases by and or or is straightforward and translates directly into the corresponding Boolean operations, i.e. and into intersection and or into union, the coordination of nominal phrases (proper nouns, common nouns) appears to pose some problems. A rather elegant theory of constituent coordination covering all kinds of linguistic categories and noun phrases in particular was proposed by Keenan and Faltz (1985). But this theory makes use of higher order denotations. The question is whether one can come up with an adequate theory for noun phrase combinations under the assumption of the very restricted hierarchy of RG.

One of the problems of traditional logic was the analysis of individual sentences: since sentences were classified into the four categoricals, the question was whether individual sentences are universal or particular. In RG, a proper noun denotes a singleton set. For singleton sets, both universal and particular return the same result, so it does not matter which operation is adopted.

Let us now look at the simple sentence

(6)  John and Mary are ill.

In transformational grammar, it has been proposed that this sentence be derived from the coordination of two sentences by so-called conjunction reduction:

(7)  John is ill and Mary is ill.

Although (6) and (7) are semantically equivalent, they are syntactically different. A semantic theory should be able to give some explanation for this fact rather than hide this fact under the term of stylistic variance. Moreover, if a semantics is claimed to fit 'hand in glove' with the structure of English sentences, as is claimed in Suppes (1982), it should be able to provide a semantic structure that is homomorphic to the structure of (6).

Assume that John and Mary form a constituent. What would be a semantic function deriving a denotation for it from the constituents denotations? For and intersection comes to mind first. However, intersection applied to two singleton sets will return the empty set, unless
proper nouns denote the same individual. Another option is union:

\[(8) \quad \text{John and Mary} = \{j\} \cup \{m\}.\]

To account for natural language and by union rather than intersection is indeed in line with the original proposal by Boole (1854). If \(I\) is the denotation for ill, (12) can be called true iff the following condition holds:

\[(9) \quad \{j\} \cup \{m\} \subseteq I.\]

Notice that we now cannot replace universal quantification by existential quantification, since both return different results for non-singleton sets.

Proper nouns cannot only be combined by and but also by or: the problem is which Boolean operation could serve the purpose to return the set denoted by John or Mary. Recall that intersection cannot be used and union has already been used in the case of and. Since and-coordinations and or-coordinations have a different meaning, one would expect that the corresponding operations should differ. It may be interesting in this context to note that the case has been made by Faltz (1989) for or being of higher procedural complexity than and.

It may be instructive here to take a closer look at Suppes' solution to computing the truth-value of categorical sentences: categorical sentences are alike in that they are made up of two absolute terms but are different with respect to the computation of their truth-value: the semantic function computing the truth value for a sentence with all is different from the semantic function computing the truth value for a sentence with some. Since all is related to and and some is related to or, one might suggest the exploitation of the different mode of combination along the following lines: let a coordination of proper nouns denote the union of their constituent denotations. Make a syntactic distinction between and-coordinations and or-coordinations of proper nouns. The grammar could then look like this:

\[(10) \quad CPNP \rightarrow PN + \text{and} + PN \quad [CPNP] = [PN] \cup [PN] \]
\[APNP \rightarrow PN + \text{or} + PN \quad [APNP] = [PN] \cup [PN] \]

where \(CPNP = \) conjunctive proper noun phrase, and \(APNP = \) alternative proper noun phrase. This allows us to have different operations to compute the truth value from \(PN\)-combinations and their predicates:
universal quantification for \textit{and}-combinations and existential quantification for \textit{or}-combinations:

\begin{align*}
(11) \quad S & \rightarrow CP\text{NP} + VP_{pt} \quad [S] = [\mathcal{F}[CP\text{NP}] \subseteq [VP_{pt}]] \\
S & \rightarrow AP\text{NP} + VP_{sg} \quad [S][\mathcal{F}[AP\text{NP}] \cap [VP_{sg}] \neq \emptyset]
\end{align*}

where \(sg\) and \(pl\) refer to the features of singular and plural respectively, and the Frege function \([\mathcal{F}]\) is defined as follows:

\[
[\mathcal{F}\phi] = \begin{cases} 
D & \text{if } \phi \text{ is true in the model,} \\
\emptyset & \text{otherwise.}
\end{cases}
\]

The solution works for elementary cases since \textit{John and Mary are ill} is true iff \(\{j, m\} \subseteq I\) and \textit{John or Mary is ill} is true iff \(\{j, m\} \cap I \neq \emptyset\). The extension of this analysis to coordinated proper nouns in object position is completely analogous. But one does not feel satisfied with the solution since a semantic distinction has somehow been solved by some syntactic trick. The shortcomings become obvious if one tries to extend this solution to the following cases:

\begin{align*}
\text{John or Bill and Mary sing} \\
\text{John and all boys sing} \\
\text{John and some boys sing} \\
\text{John or all boys sing} \\
\text{John or some boys sing}
\end{align*}

Any conventional semantic analysis that has a category \textit{NP} and subsumes the category of proper nouns under this category would be able to compute the denotation of the sentences from the respective constituents \textit{NP} and \textit{VP} rendering, e.g., \textit{John and some boys} into an \textit{NP}-constituent. No such analysis is available in RG, though. It is likely that we can come up with some local solution for each case, but at the same time fail to grasp the relevant generalizations that follow from having a category \textit{NP}.

Now consider the sentence

\begin{equation}
(12) \quad \text{John likes Mary and Bill Susan.}
\end{equation}

Structures like these have become known under the term 'gapping', since they can be conceived of as being derived from a conjunction of two sentences

\begin{equation}
(13) \quad \text{John likes Mary and Bill likes Susan}
\end{equation}
by some syntactic transformation that leaves a gap where the verb should have occurred for the second time. If (6) is generated from its constituents why not do the same for (12)? Notice that (12) need not be considered to be derived from a conjunction of sentences but can equally well be considered as a coordination of a sentence and a pair of proper nouns. The question is how to provide a semantic structure for (12) where $L$ enters only once in the structure. A possible candidate is:

$$L \cap ((\{j\} \times \{m\}) \cup (\{b\} \times \{s\})).$$

Two points may be raised against this proposal: first, (14) is not an expression of an extended relation algebra for the reason that the Cartesian product cannot be defined in terms of the fundamental operations of an extended relation algebra (1). Second, (14) is not in line with the analysis of other sentences: the same phrase *likes Mary* occurs as a constituent of *John likes Mary* but does not occur as a constituent of the semantic structure of (12), if we accept (14) as the semantic structure of (12). It is not plausible that one and the same phrase occurs as a constituent in one structure but not as a constituent in a similar structure.

A similar problem arises with the structure

$$\text{(15)} \quad \text{John likes Mary and Bill too}$$

that has at least the two readings

- *John likes Mary and Bill likes Mary*
- *John likes Mary and John likes Bill*

depending on the prosodic features with which the above sentence is uttered. The problem is which structures can be provided for (15) that are homomorphic to its syntactic structure.

3. NUMBER

According to the standard set-theoretical definition, a cardinal number $n$ denotes the set of all sets having $n$ elements. This denotation is not within the limits of the set-theoretical hierarchy (2). As one can almost predict, RG will have trouble analyzing natural language constructions that somehow involve the notion of number. The notion of number occurs in a variety of contexts in natural language, like expressions with
cardinal adjectives, e.g., *ten books*, or the morphological category of number exhibited by the distinction between singular *book* and plural *books*.

**Cardinality**

Suppes (1974) proposed for the phrase *two red flowers* the following structure:

\[(16) \quad 2 \cap \mathcal{P}(R \cap F)\]

where \(R\) denotes the set of red objects and \(F\) the set of flowers. This solution is not feasible in RG, since it falls outside (2).

Assume we think of numbers as a set of basic entities that fulfil Peano’s axioms. But then the following problems arise: (i) what would the denotation be for *two red flowers*?; and (ii) how is this denotation derived from constituent denotations 2, \(R\), and \(F\)?

**Collectivity**

The conjunction *and* can be construed by intersection in verbal contexts and by union in nominal contexts as we have just seen. There are however cases where *and* cannot be construed as a Boolean operation:

\[(17) \quad \text{Mary and Susan are sisters.}\]

Predicates such as those occurring in (17) are conventionally referred to under the term ‘collective’. Collectives therefore pose a problem to relational grammar. A way out could be along the lines of Link (1983) where collectives occur as elements of the domain \(D\). However, each model structure would then have to be enriched by a relation between collective individuals and the non-collective individuals occurring as their parts, since (17) implies

\[(18) \quad \text{Mary has a sister.}\]

If the denotations of both *Mary* and *Mary and Susan* occur as elements of the domain then there is no way to make the inference from (17) to (18) explicit.

**Singular and Plural**

The natural language fragments presented by Suppes so far are in the singular if dealing with proper nouns and in the plural if dealing with
common nouns as occurring in categorical sentences. One would like to know whether singular and plural can be given a unified analysis in relation algebraic semantics. What is desired is a grammar that derives semantic trees for *All (some, no) dogs bark* and *Every (some, no) dog barks* at the same time.

The set-theoretically straightforward procedure would be to have the singular denote the set of all one-elemented sets

$$\text{Singular} = \{ X \subseteq D \mid X \text{ is an atom of } \mathcal{P}(D) \}$$

and have the plural denote the set of all sets with more than one element

$$\text{Plural} = \mathcal{P}(D) - (\text{Singular} \cup \{\emptyset\}).$$

One could then define:

$$[\text{girl}] = \mathcal{P}(G) \cap \text{Singular}$$
$$[\text{girls}] = \mathcal{P}(G) \cap \text{Plural}$$

where $G$ is the set of girls of $D$. However, this is not feasible due to the restriction to the set-theoretical hierarchy in (2). It is unclear how this difference can be accounted for in RG.

Notice that there are semantic differences between singular and plural.

(19) *A limousine was provided for all guests*
(20) *A limousine was provided for every guest*

(19) has at least a collective reading whereas (20) has at least a distributive reading.

**Bare Plurals**

Bare plurals pose a problem to linguistic analysis, since they have to be construed by universal quantification on some occasions and by existential quantification on other occasions. In Suppes (1979) bare plurals in *there are*-structures are analyzed by existential quantification: the sentence *There are trees* is given the same denotation as the sentence *There are some trees*. In Suppes (1976) bare plurals in object position are also analyzed by existential quantification: the verb phrase *date juniors*
is given the same denotation as the verb phrase date some juniors. On the other hand, sentences such as Birds are smaller than foxes, as occurring in Schubert’s steamroller (cf. Purdy, 1992), require construal by universal quantification in both their subject and object positions.

4. PRONOUNS

The use of variables in logical language has been considered as an analogue of the use of (anaphoric) pronouns in natural language. The correct semantic analysis of pronominal constructions should therefore be a touchstone for any variable-free approach. Some cases were examined in Böttner (1992a). The sentence

(21) Bill loves his children

can be called true if

(22) \{b\} \cap -dom(\bar{C} \cap -L) \neq \emptyset

holds where L is the relation of loving, C is the relation of being a child of, and

\[ dom(R) = \bar{R}_x D. \]

Notice that

\[ -dom(\bar{C} \cap -L) = \{x \mid (\forall y)(yCx \rightarrow xLy)\} \]

i.e., it denotes the set of all x, such that if y is x’s child then x loves y. We therefore have the following denotation

(23) \[ [\text{loves his children}] = -dom([-\text{love}] \cap [\text{child}]^\cup). \]

More complicated is it to find a denotation for

(24) Bill loves his children and so does Harry.

It has at least the following two readings:

(25) Bill loves Bill’s children and Harry loves Harry’s children

(26) Bill loves Bill’s children and Harry loves Bill’s children.
A third reading may be provided if the possessive pronoun refers to someone mentioned outside the sentence in question. A paraphrase of the reading (25) of (24) is

\[(27) \quad \text{Bill and Harry love their children.}\]

This reading can easily be accommodated in RG using the framework developed in a preceding section. There are the following problems: (i) how can the structurally different sentence (24) be mapped onto the structure denoted by (27)?; and (ii) how can the reading (26) of (24) be correctly spelt out in RG? What is needed here is some semantic operation corresponding to the and so does-construction.

A notorious problem for semantics is posed by so-called donkey sentences like, e.g.:

\[(28) \quad \text{Any man who owns a donkey beats it.}\]

The problem with this type of sentence is that it refers to some object that is mentioned in the embedded relative clause. (28) has man who owns a donkey as a constituent. The semantic structure of this constituent is

\[(29) \quad M \cap \bar{O}“D\]

where \(M\) is the set of men, \(O\) the relation of owning and \(D\) the set of donkeys. But there is no way to embed (29) into a sentence. The structure

\[(30) \quad (M \cap \bar{O}“D) \subseteq \bar{B}“D\]

does not adequately construe the meaning of (28) but rather of

\[(31) \quad \text{Any man who owns a donkey beats a donkey}\]

which is not equivalent to (28).

An interesting type of pronoun is exemplified by the words same and different as occurring in

\[(32) \quad \text{John and Mary read the same books}\]

\[(33) \quad \text{John and Mary read different books.}\]

They have been analyzed as a species of quantifiers by Keenan (1992) and van Benthem (1989). The interesting thing is that they cannot be
construed by standard monadic (or Fregean) quantifiers. The question is whether RG fares any better than quantifier logic in this case. Notice first that there can be set-theoretical structures provided for (32) and (33):

\begin{align*}
(34) & \quad B \cap R^\{j\} = B \cap R^\{m\} \\
(35) & \quad B \cap R^\{j\} \neq B \cap R^\{m\}.
\end{align*}

Notice second that (34) and (35) do not admit of a structure parallel to read all/some/no books where read the same books or read different books is a phrase of category VP. (Such an analysis was tentatively given in Böttner (1992a) but not within the confines of RG.) Notice third that no similar structure appears to be available for

\begin{align*}
(36) & \quad \text{All students read the same books} \\
(37) & \quad \text{All students read different books}
\end{align*}

5. VERBS

Whereas the restricted hierarchy of denotations may be appropriate for nouns and noun phrases, it is certainly less appropriate for verb phrases. Verb phrases generally bear a tense specification. Moreover, verb phrase denotations exhibit a wide variety of semantic types like states, events, processes, actions. It is certainly not accidental that the examples of verb phrases of the various English fragments presented so far are in the simple present tense and refer to properties, like are sick, are students, states like, e.g. loves Mary, like some teachers, are in the house, or habits like, for example, bite people, eat some vegetables, date all juniors. The question remains how tensed predicates like, were sick, will be sick or predicates that do not just express a property or a state, e.g. are eating vegetables, open some windows, bake a cake, etc., can be accommodated in RG.

It should be mentioned, though, that Suppes’ own position towards RG has not been without reservation. In Suppes (1980) and, in particular, in Suppes (1979), the view is expressed that RG may turn out to be too weak for the purpose envisaged and it is argued for enriching RG.
by procedures. In fact Suppes has been arguing for using procedures in
natural language semantics from as early as 1973 onwards, cf. Suppes
(1973b), and has always emphasized that set theoretical semantics is
some useful abstraction from underlying procedures. A first detailed
proposal for the enrichment of RG by procedure is given in Crangle
and Suppes (1987) and in Suppes and Crangle (1988) under the term of
‘context-fixing semantics’ (CFS).

CFS is designed for a fragment of arithmetical instruction. The
model structures are therefore arrangements of symbols in rows and
columns. Since symbols are the entities of the domain $D$, the denotation
of column to the left will be a set of such objects and therefore be higher
up in the denotational hierarchy. The denotation of the adjective top as
in, e.g. top number or top spot is defined

$$\lambda RS \{a \mid a \text{ is the } R\text{-last element of } S\}$$

where $S$ is some set ordered by some ordering relation $R$ and $a$ is called
the $R$-last element of $S$ iff $a \in S \land (\forall b)(b \in S \rightarrow (b = a \lor bRa))$.
(38) denotes a function that takes a pair $(R, S)$ as its argument where
$R$ is an ordering relation on $D$, $S$ is a subset of $D$, and returns a subset
of $D$. Since $(R, S) \in \mathcal{P}(D \times D) \times \mathcal{P}(D)$, the domain of (38) is
$\mathcal{P}(\mathcal{P}(D \times D) \times \mathcal{P}(D))$. Since the domain of this function falls outside
RG, the function denoted by (38) falls outside RG, too. Therefore, it
appears that a grammar with a CFS cannot be an RG.

A major problem that has not been solved in any of the procedural
extensions of RG is the meaning of an action. An action may effect
a change of the model structure either in its domain (by bringing new
objects into existence) or in its valuation function or in both. The general
scheme for a verb phrase $\alpha$ denoting an action would thus be

$$[\alpha] = \{\langle D, D' \rangle : \phi(D, D')\}.$$  

In order to illustrate this type of verbal denotation let us consider a
simple model structure of an arrangement of digits and letters on a line
like this:

$$(40) \quad a \ 1 \ b \ 2.$$  

(40) can be described as a set of objects $D$ ordered by some strict linear
relation $L$, i.e. $L$ is a binary relation on $D$ that is transitive, asymmetric
and connected. The action denoted by the verb phrase

$$(41) \quad \text{remove all digits}$$
applied to (40) leads to the new model structure

\[(42) \quad a \ b.\]

The action denoted by (41) has the effect of changing the model structure (40), since the domain \(D\) got changed, the denotation of digit is different, as is the denotation of left and the denotation of every notion that depends on this relation. This change effected on the model structure (40) can be represented as follows:

\[(43) \quad \begin{align*}
D & \quad \{a, b, 1, 2\} & D' & \quad \{a, b\} \\
digits & \quad \{1, 2\} & digits' & \quad \emptyset \\
letter & \quad \{a, b\} & letter' & \quad \{a, b\} \\
left & \quad \{(a, 1), (1, b), (b, 2), (a, b)\} & (a, b), (a, 2), (1, 2). & \\
\end{align*}\]

It is easy to compute output sets from input sets. They may not be affected at all as in the case of the denotation of letter

\[T' = T\]

or may easily be derived as in the case of the denotation for digit

\[G' = G - G.\]

More difficult is the derivation of the output relation for \(L\). The procedure that turns \(L\) into \(L'\) would have to remove from \(L\) all pairs that have a digit occurring in it:

\[L' = L \cap (-X \times -X).\]

As an instance of (39) we propose

\[(44) \quad \begin{align*}
[\text{remove}] = \{ & \langle (D, X), D' \rangle : D' = D - X \land \\
& v' = v \mid D' \land \\
& L' = L \cap (-X \times X) \}
\end{align*}\]

But this denotation certainly exceeds the limits of RG.

6. CONCLUDING REMARK

This paper is meant to contribute to a festschrift. A festschrift is some sort of a birthday present, and a birthday present is supposed to come as
a surprise. In this sense, by contribution is not a present, since most of the issues mentioned have been topics of numerous conversations and much correspondence with Pat over the last decade. I would not have brought them up in the present context had I not the strong conviction that Pat will come up with some brilliant and surprising ideas for their solution.

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REFERENCES


COMMENTS BY PATRICK SUPPES

Michael Böttner has given an excellent, detailed survey of problems that remain open for the use of relational grammars in the analysis of the semantics of natural language. Reading his paper is, for me, like having an extended conversation with him. Most of the issues he writes about are ones that we have discussed, in most cases more than once. This is not the right occasion to enter into the necessarily complicated details of
proposing new solutions to the various open problems he poses. What I think is more appropriate is to comment on some of the conceptual matters surrounding the development of relational grammars and their applications.

**Skepticism about First-Order Logic as Logical Form.** The motivation for my own work in variable-free semantics, as reflected in relational grammars, is skepticism of the tendency of philosophers to claim that logical form should be expressed in the notation of first-order logic. As a psychological thesis about the way we internally compute either the production or comprehension of natural-language utterances, I find this proposal almost certainly the wildest of fairy tales. There is, it would seem to me, no serious evidence of any kind that we process natural language in a way that depends at all on such a logical form. Certainly there can be some usefulness in formal analysis in terms of first-order logic, but not as a serious thesis about the syntactic or semantic structure of natural language. Given our difficulty of learning second languages as adults, it seems likely that a great deal of our individual natural languages are extensively used in our internal storage, generation and analysis of natural language utterances. In other words, the internal computing mechanisms that a speaker of English stores are in detail quite different from those stored by a speaker of Chinese or Japanese, which also differ from each other considerably. This does not mean that the basic mechanisms of language learning in the child are fundamentally different, but it does mean that the end result is different.

Another reason for skepticism about first-order logic is that the analysis of sentences of a natural language in terms of first-order logic only works for what I like to call ‘classroom English’, and I am sure the situation is similar for other natural languages. Once we go to the robust give-and-take of street talk or any other informal situation of linguistic communication, the resources of first-order logic are much too awkward and limited to carry us very far. I also hasten to add that some of these difficulties are also shared by relational grammars, which have as an objective the direct mirroring of the syntactic structure of a natural language. What happens in the case of relational grammars in dealing with rapidly moving dialogue is that a good deal of the syntax can be represented in some form, even if the semantics remains obscure. I tried to bring out the resistance of much ephemeral language use to systematic semantic analysis in my discussion of the Valley Girl phrase *grody*
to the max (Suppes, 1984, pp. 170–172). Those with strong systematic foundational views of language will be, it seems to me, necessarily frustrated in their attempts to give a universal semantic framework for the analysis of natural language. Such universal foundational theses are as hopeless here as in other domains of experience. My ambitions for relational grammars are not so imperialistic. I only hope that they will be able to solve a certain range of problems that will throw light on more systematic uses of ordinary language, if not the cases of grody to the max and its ilk.

My own view, undoubtedly mistaken in all kinds of details, to say the least, is driven by the view that human memory is much more capacious than computations are fast. So I think of natural language as being represented internally by a vast array of shallow trees representing many different kinds of grammatical forms. Much of what is going on can be nicely represented in context-free grammars with a direct semantic interpretation, although more troublesome cases require extensions to generalized phrase structure grammars. But for computational reasons they are not extended to transformational grammars which, unless severely restricted, are undoubtedly computationally too intensive.

**Semantics of Actions.** Michael rightly remarks that relational grammars, or even their procedural extensions, do not give a very adequate account for the meaning of an action. In fact, in the general approach it is hard to distinguish actions from spatial relations, let us say in the case when they are both represented as binary relations. I am not sure what will be the best way to move toward a deeper and better analysis, but I do have some thoughts and conjectures, especially generated by my recent work on machine-learning of natural language with Michael and Lin Liang. At the top level, in the case of the computer as the natural language user, we have a LISP representation internally, and at this level of abstraction there is nothing special about the representation of actions. At this level of computing, so to speak, an action is not much different from a spatial relation. At the level of LISP, the dominant position of actions in commands shows up only in the structure of the trees, not locally in the nature of the lexical treatment. However, this changes rapidly as we move to the actual details of robotic implementation or analysis of how the human perceptual-motor system works. The overwhelming difference between actions and spatial relations as we move downward in detail is that the actions are connected to the motor
systems as well as the perceptual system. These connections can, of course, be reflected themselves at different abstract levels, but it is surely their existence that is one of the most essential marks of the difference. In the standard treatments of the semantics of commands, perceptual and motor control details do not enter in an explicit and formal way, for the semantic analysis is ordinarily left at a high level of abstraction. Even in the case of quite simple robots, we find that in the programming structure there are at least three or four levels of abstraction above the very concrete control of the robot, before we reach the top level which has the standard LISP representation.

There is no natural bottom to this search for level of detail, and this is surely especially true when we are dealing with the human perceptual and motor systems. The level of detail for semantic or other purposes will need to be fixed by the aims of the investigation, in other words, by the problems that are the focus of attention. In machine-learning of natural language it is obvious that much of the work can take place as far as robotic learning is concerned working only with top-level LISP representations. Obviously this is quite inadequate for other purposes. For example, the learning of concrete concepts in conjunction with language learning. In what we call ‘pure language learning’ where the concepts used are already assumed available to the robot, such mixing of concept learning and language learning does not occur and a very high level of representation can be adequate.

This is the point with which I want to end. It is at this high level of representation that the relational grammars and the accompanying semantics in terms of extended relation algebras can be quite useful. As we move to greater detail such grammars will be of less use, but there is some reason to hope that the right way to think about perception and motor systems is itself pluralistic in terms of a very considerable degree of decentralization. Communication with the peripheral systems can be conceptualized as being in a stripped-down and relatively meager language. The richness of natural language as we think of it is not used for this communication and is not understood even in an approximate form by the peripheral systems. If this way of looking at things is even roughly correct, it justifies separating a large piece of the linguistic analysis from the details of perception and motor-control, especially in terms of what resides in the peripheral systems which range from the fingertips to the eyeballs, or in the case of a robot, from the grasper to the television camera.
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