CHARMING PROBABILITY JUDGMENTS

Charles Peirce insisted a long time ago that justifying beliefs currently held is unnecessary. Only changes in belief require justification. Epistemologists should turn away from the question of justifying what they already believe and focus instead on when and how rational inquirers should change their minds.

Patrick Suppes addressed the topic of change in belief in his 1965 critique of updating credal probabilities by conditionalization (Suppes, 1965, Ch. 3). The following remarks reflect preoccupations prompted in part by Suppes’s ideas. I have always been grateful for the stimulus.

In order to speak systematically of rational belief change, we need some way of representing the changes we are talking about. I find it congenial to identify a space of potential cognitive states and to represent changes in belief as shifts from one such state to another. A caveat must be entered when using this approach. There is no scheme of potential cognitive states which can capture those changes which are the product of conceptual innovation; for such innovation typically involves an enlargement and, hence, a change in the space of potential cognitive states itself rather than a change within a given space. But once we obtain a systematic purchase on the structure of a space of potential cognitive states we also gain entry into the investigation of changes in the particular conceptual space we are using. For example, we begin to appreciate not only the possibility of conceptual changes which are cases of conceptual innovation but also of conceptual destruction. My concern here, however, is not with conceptual change. I shall suppose that we are given a fixed conceptual framework or space of potential cognitive states. I shall be talking about change in credal probability judgement. Prior to doing that, however, I want to specify which aspect, if any, of a cognitive state is changed when probability judgements are altered.

Among those who think (as both Suppes and I do) that states of credal probability judgement (credal states) are determined by cognitive states, there is a difference of opinion as to whether a credal state is identical to a cognitive state (so that the determination is trivial) or whether, a cog-
nitive state has two independently varying components: a state of full belief and a rule (a ‘confirmational commitment’) specifying for each potential state of full belief what the credal state should be. According to this second view, credal states are determined by the agent’s current state of full belief and current confirmational commitment – i.e., by the two components of the cognitive state. Suppes (1965, p. 61) and many others endorse the former position. I resist it.

Credal probability judgements are of interest primarily in cases where inquiring and deliberating agents regard the doxastic propositions to which probabilities are assigned as both possibly true and possibly false. But as de Finetti rightly observed, the notion of possibility appropriate here is epistemic or serious possibility – i.e., consistency with a state of full belief (De Finetti, 1974, 24-32). In this sense credal states presuppose states of full belief. Consequently states of full belief are distinct attitudinal states from credal states.

It may be replied (following observations made by Suppes) that according to the requirements imposed on coherent credal states, consequences of states of full belief receive credal probability 1. Since a state of full belief may be characterized by its consequences, it follows that a state of full belief can be adequately represented by the part of the credal state assigning probability 1 to propositions (Suppes, 1965, p. 61.)

There are at least two reasons why this suggestion ought to be resisted.

In the first place, there can be serious possibilities relative to the agent’s state of full belief assigned credal probability 1 whose complements are also serious possibilities (De Finetti, 1974, 3.11; Levi, 1989). An inquirer may be ‘almost certain’ that tossing a coin until it lands heads the first time will terminate in a finite number of tosses without being certain. A customer shopping in the supermarket may be almost certain that chicken he contemplates purchasing does not weigh precisely 5 pounds without being absolutely certain. To be almost certain is a far cry from full belief. The customer could be almost certain that the chicken does not weigh exactly \( m \) pounds for every real value for \( m \) between 0 and infinity while assigning probability 0.95 to the claim that it weighs approximately 5 pounds (say 5 pounds give or take an ounce). In this sense, the credal state does not discriminate between doxastic propositions which are fully believed and those judged possibly false.
but which get credal probability 1. A credal state does not determine a state of full belief without extra provisos.¹

But even if we waive this point, several distinct coherent credal states may yield the same state of full belief. If we are prepared to consider the inquiring agent as committed at a given moment to some rule for judging which credal states are appropriate to which potential states of full belief, we are, in effect, acknowledging that the inquiring agent has either tacitly or explicitly a confirmational commitment.

Given the idea of a confirmational commitment, we may then characterize a cognitive state as consisting of two independent components: a state of full belief \( \mathbf{K} \) and a confirmational commitment \( C \) which is a function from potential states of full belief to credal states. The current credal state \( \mathbf{B} \) is then understood to be identical with \( C(\mathbf{K}) \).

Someone may object that this way of characterizing cognitive states and their relations to credal states begs too many controversial issues to be acceptable as the basis for a discussion of approaches to rational change in credal probability judgement.

Thus, appealing to confirmational commitments may be thought to entail endorsing views of 'logical' probability judgement favored by authors like Keynes (1921), Jeffreys (1957, 1961) and Carnap (1962). For example, Carnap's confirmation functions are functions from potential states of full belief to states of credal probability judgement. They are potential confirmational commitments. Carnap, however, sought to identify a standard confirmational commitment which every rational agent ought to adopt. Such a standard would meet the requirements for a logical probability or a logical confirmation function.

However, one need not embrace the necessitarian views of authors like Keynes, Jeffreys and Carnap in order to acknowledge confirmational commitments as components of cognitive states. One can deny the existence of a fixed, standard, 'logical' confirmational commitment which everyone meeting minimal requirements of rationality should satisfy.

Nor need it be the case that once an agent has endorsed a confirmational commitment, the agent is committed to it forever. States of full belief are subject to critical review and alteration.² So are confirmational commitments. Indeed, it would be absurd to think otherwise once one has conceded that there is no fixed standard confirmational commitment that every rational agent should endorse.
It may be objected that by adopting such a view, we are begging the question against views which deny that the rational agent should have by his or her own lights any judgement as to what his or her credal state should be given his or her current state of full belief. This complaint would be just were credal states required to be numerically determinate. But once indeterminate credal states representable by (convex) sets of numerically definite probability functions are allowed, an agent who is said to have no judgement as to what his credal state should be given his or her current state can be glossed as having no judgement as to what numerically determinate credal state he should have given his current credal state (Kyburg, 1961; Levi, 1974).

Perhaps a critic of the idea of using confirmational commitments will point out that confirmational commitments are useful only if there are interesting occasions where states of full belief change while confirmational commitments remain constant. Under such circumstances, confirmational commitments can be used to derive a new credal state. If no such circumstances ever obtain, we may continue to represent cognitive states as constituted out of states of full belief and confirmational commitments; but we could more simply proceed to account for changes in credal state without reference to confirmational commitments which will then appear always to spin their wheels.

Even if we grant this point, it should be noted that by representing cognitive states by ordered pairs consisting of states of full belief and confirmational commitments, we are not presupposing in advance that confirmational commitments are useful. Sceptics can remain sceptics on that point. On the other hand, if we dispensed with the use of confirmational commitments at the outset and focused directly on changes in credal state, we would be begging questions against views which (under some title or other) take confirmational commitments seriously. Thus, representing cognitive states as having two independent components, states of full belief and confirmational commitments, which can be varied independently, permits discussion of a wide variety of differing views concerning change in credal state without begging controversial issues at the outset. 3

Insofar as we represent a potential state of full belief $\mathbf{K}$ in the conceptual space of potential states $\mathcal{K}$ by a set of sentences $\mathcal{K}$ in language $\mathcal{L}$ closed under consequence (i.e., by a ‘theory’ or ‘corpus’), a credal probability relative to $\mathcal{K}$ will be a coherent probability measure relative to $\mathcal{K}$ – i.e., a probability measure defined over sentences in $\mathcal{L}$ where
all members of \( \mathcal{K} \) are assigned unconditional probability 1 and two sentences equivalent given \( \mathcal{K} \) are assigned the same probability.

A credal state relative to \( \mathcal{K} \) is a set \( \mathbf{B} \) of probability measures.

A probability function \( Q \) is permissible according to \( \mathbf{B} \) if and only it is a member of \( \mathbf{B} \).

A credal state is convex if and only if for every pair of permissible functions \( Q \) and \( Q' \), every sentence \( \mathcal{E} \) in \( \mathcal{L} \) consistent with \( \mathcal{K} \), and for every positive real \( \alpha \) less than or equal to 1, \( \alpha Q(\cdot/F) + (1 - \alpha)Q'(\cdot/E) \) is permissible according to \( \mathbf{B} \).

If \( \mathbf{B} \) is a coherent set of probabilities relative to \( \mathcal{K} \), \( \mathbf{B}' \) is a coherent set relative to \( \mathcal{K}' \) and \( \mathcal{K}' \) is the expansion of \( \mathcal{K} \) by adding \( \mathcal{E} \) to \( \mathcal{K} \) and forming the logical closure, \( \mathbf{B}' \) is the conditionalization of \( \mathbf{B} \) if and only if (a) for every \( Q \) in \( \mathbf{B} \) there is a \( Q' \) in \( \mathbf{B}' \) and (b) for every \( Q' \) in \( \mathbf{B}' \) there is a \( Q \) in \( \mathbf{B} \) such that \( Q'(\cdot/F) = Q(\cdot/F\&\mathcal{E}) \).

A confirmational commitment is a function \( C \) from potential states (or the theories representing them) in the conceptual space \( \mathcal{K} \) to credal states in \( \mathbf{B} \). A cognitive state consists of a belief state \( \mathbf{K} \) and a confirmational commitment \( C \).

I assume that confirmational commitments satisfy the following rationality conditions:

**Coherence:** If \( Q \) is permissible according to \( C(\mathcal{K}) \), it is coherent relative to \( \mathcal{K} \).

**Consistency:** If \( \mathcal{K} \) is consistent (i.e., distinct from \( \mathcal{L} \), \( C(\mathcal{K}) \) is nonempty.

**Convexity:** \( C(\mathcal{K}) \) is convex.

**Confirmational Conditionalization:** If \( \mathcal{K}' \) is the expansion of \( \mathcal{K} \) by adding \( \mathcal{E} \) and forming the closure, \( C(\mathcal{K}') \) is the conditionalization of \( C(\mathcal{K}) \).

All of these requirements have been controverted by someone or other but none of them is rejected by classical strict Bayesians. Indeed, a strict Bayesian insists on imposing yet another condition:
Uniqueness: For every consistent \( \mathcal{K} \), \( C(\mathcal{K}) \) is a unit set.

Uniqueness implies convexity and consistency. Strict Bayesianism can be characterized, therefore, as the view which requires coherence, uniqueness and confirmational conditionalization.

Credal uniqueness is unacceptable as a condition on rational probability judgement. It insists that rational agents adopt confirmational commitments recommending for every potential state of full belief a single probability measure as uniquely permissible. There is no room for the agent to be in doubt as to what his credal probability judgements should be in a given state of full belief.

Credal states may change due to a change in state of full belief (or corpus) or due to a change in confirmational commitment. If the confirmational commitment is held fixed, then an expansion of the initial belief state requires that the new credal state be the conditionalization of the old. A contraction of the initial belief state requires that the old credal state be the conditionalization of the new. The first kind of change of credal state is often called conditionalization. I call it temporal credal conditionalization to contrast it with the atemporal requirement of confirmational conditionalization and with inverse temporal credal conditionalization which is the second kind of change just mentioned.

Notice that if the confirmational commitment is held fixed and there is no change in the state of full belief, there should be no change in credal state.\(^4\)

Whether changes in credal state are due to changes in the state of full belief (= evidence = background information) or due to changes in the confirmational commitment, the agent’s credal state is uniquely determined at any given time by his or her state of full belief or total available evidence and his or her confirmational commitment. That is to say, \( C(\mathcal{K}) = B \). This equality is a succinct expression of the total available evidence requirement. The total evidence requirement rests on the following assumptions: (i) A cognitive state is representable by independently variable components: the state of full belief and the confirmational commitment. (ii) The confirmational commitment is a function from potential states of full belief to credal states. Once these assumptions are accepted, the total evidence requirement is the trivial consequence that given the current state of full belief and confirmational commitment, \( C(\mathcal{K}) = B \).
Suppes (1965, p. 60) suggested that the total evidence requirement could be captured by considering the current credal state alone; for that credal state, if coherent, would determine which hypotheses receive credal probability 1. But we have already seen why that suggestion will not do. Not every claim receiving credal probability 1 is a consequence of the current state of full belief. Moreover, the total evidence requirement presupposes that the agent's cognitive state is representable by two independently variable components, a state of full belief and confirmational commitment, and that the credal state is uniquely determined by these.5

One might reduce the thesis that the credal state is determined by the state of full belief and the confirmational commitment to triviality by maintaining that all changes in credal state involve changes in confirmational commitment. But Suppes does not do so. He concedes that sometimes credal states are altered via temporal credal conditionalization (p. 60). This is tantamount to claiming that some changes in credal state involve changes in full belief without change in confirmational commitment. To this extent, Suppes is conceding that there is a useful role for a total knowledge requirement over and above what coherence conditions require. That role is captured in the claim that the credal state is determined by the state of full belief and confirmational commitment.

Advocates of confirmational tenacity maintain that once rational agent $X$ has endorsed a confirmational commitment, he or she should never revise it (except, perhaps, when a change in conceptual framework is contemplated). Those who impose this requirement on changes in credal state can only countenance two kinds of changes: temporal credal conditionalization when the state of full belief is expanded and inverse temporal credal conditionalization when it is contracted. In practice, attention has been focused on temporal credal conditionalization.6

Authors like Keynes, Jeffreys and Carnap sought to construct confirmational commitments which could either be proposed as permanently fixed standards for all rational agents or, if change is allowed at all, is allowed only under special circumstances where the change could be determined in an objectively controllable way. That advocates of the use of standardized confirmational commitments should favor confirmational tenacity at least pending the time when a change in standard is legitimately called for is not too surprising. In point of fact, however, serious efforts at giving accounts of when and how to modify confirmational commitments are pioneered in the writings of Carnap
(1952). Ironically, the topic of revising confirmational commitments is ignored by personalists like de Finetti and Savage. This neglect would not be surprising if these authors and their epigones had recognized the legitimacy of changing credal states in violation of temporal credal conditionalization or its inverse in arbitrary ways (provided that coherence is preserved). They did not, however, do so. Savage, in point of fact, sought to fend off the complaint that personalism supported excessive probabilistic anarchy by showing that under appropriate conditions persons who differed in their prior credal states could approach a kind of consensus through updating via Bayes theorem given sufficient data (Savage, 1954, 3.6, 4.6). To achieve this result, Savage presupposed that the inquirers agreed at least to obey temporal credal conditionalization and, to that extent, to conform to confirmational tenacity. Without this assumption, Savage’s argument fails.

Yet, there is no principled reason available to Savage or other personalists as to why confirmational tenacity should be obeyed. At any given time $t$, any credal state coherent with the state of full belief at that time is acceptable even if the shift to that credal state reflects a violation of confirmational tenacity.

Suppes appreciated that confirmational tenacity is in trouble quite clearly in his 1965 paper.

It seems important to recognize that the partial beliefs, or probability beliefs as we may term them, that an individual holds as a mature adult are not in any realistic way, even for an ideally rational individual, to be obtained simply by conditionalization, that is, in terms of conditional probability, from an overall probability measure which the individual was born with or acquired early in life. The patent absurdity of this idea does not seem to have been adequately reflected upon in Bayesian discussions of these matters. The static adherence to a single probability measure independent of time is characteristic of de Finetti and Savage, but even a superficial appraisal of the development of a child’s information-processing capacities makes it evident that other processes than conditionalization are required to explain the beliefs held by a mature adult. Moreover, even an adult who does not live in a terribly static and simple world will need other processes than conditionalization to explain the course of development of his beliefs during his years as an adult (Suppes, 1965, p. 61).

Suppes’s chief reservations with this Carnapian vision (Carnap, 1962, pp. 310–311) seem to be that temporal credal conditionalization is inadequate for determining what features of the experiential input the inquiring agent receives should be taken into account in modifying his or her credal state and to question whether the data which should be retrieved are always adequately processed by means of temporal credal conditionalization (Suppes, 1965, pp. 62–65). Suppes must be
right about this. To apply temporal credal conditionalization requires expanding the initial belief state by becoming certain of additional propositions. When such expansion is the output of implementing a program for routine expansion by observation (Levi, 1980, Ch. 2; 1991, Ch. 3), which sensory inputs are processed and which are censored in implementing the program is clearly a question that does not appear amenable to a ‘bayesian’ treatment. And the expansion which results is not itself a change in state of full belief due to conditionalization.

Suppes’s perfectly correct and insightful observation establishes the point that updating by Bayes theorem via temporal credal conditionalization cannot provide a complete account of rational change in credal probability judgement because it needs supplementation by an account of how new information is added to the state of full belief through observation. But this point does not itself argue for a need to give up confirmational tenacity. Suppes calls upon us to recognize the incompleteness of confirmational tenacity but says nothing about its incorrectness. I want to insist on its incorrectness as well.

The first principle of the kind of pragmatism I am fond of advocating runs as follows:

*Cognitive Inertia:* Where it doesn’t itch, don’t scratch!

Two corollaries of this principle are relevant to our concerns.

*Doxastic Inertia:* Don’t alter a state of full belief unless one has a justification for doing so.

*Confirmational Inertia:* Don’t alter a confirmational commitment unless one has a justification for doing so.

Doxastic inertia is clearly advocated by Peirce in his ‘Fixation of Belief’. I propose extending it to confirmational inertia as well.

These principles naturally raise the question of when changes in states of full belief and confirmational commitments are justified. In this discussion, I want to discuss very briefly two types of occasion when changes in confirmational commitments may be warranted.
Consider, then, the kind of anxiety Savage sought to allay. George and Barbara disagree in their credal states with respect to some binomial parameter $p$ for a Bernoulli process. George uses the ‘flat’ prior $f(p) = 1$ whereas Barbara uses the prior $f^*(p) = 9!9! / 19! p^9 (1 - p)^9$. If George and Barbara are going to engage in a joint inquiry seeking a good estimate for $p$ and observe 50 positive results on 100 trials and both observe confirmational tenacity, George’s credal probability distribution after observation is

$$f(p; r/n = 0.5) = \frac{50!}{50!101!} p^{50} (1 - p)^{50}$$

whereas Barbara’s credal probability function would be

$$f^*(p; r/n = 0.5) = \frac{59!}{59!119!} p^{59} (1 - p)^{59}.$$ 

These densities determine credal probabilities for hypotheses specifying interval estimates for the value of $p$ which are in very close agreement. Personalists, however, have no reason to insist that George and Barbara observe confirmational tenacity.

Suppose, then, that confirmational inertia is endorsed. Will that suffice to allow George and Barbara to retain their respective confirmational commitments? Endorsing confirmational inertia will be indistinguishable from endorsing confirmational tenacity unless some circumstances can arise where revision of confirmational commitments is justified. Now, the mere existence of a difference in opinion may not always suffice to furnish such a justification. When George and Barbara respect each other’s views on the subject under discussion to the point of being concerned to resolve the dispute between them without begging any points in contention, both parties have good reason to revise their probability judgements without changing evidence. They may have good reason to change their confirmational commitments so as to recognize the probability judgements of the other as permissible. In that case, they both will begin their joint inquiry from a single consensus point of view.

Observe, however, that what they should do under the circumstances is to move into a form of suspense where their initial priors are both taken seriously. This can be done by shifting to the credal state which is the convex hull of $f$ and $f^*$. Such a credal state would represent the shared agreements of George and Barbara prior to undertaking the joint inquiry. Unless some new consideration intervenes, George and Barbara can keep the new indeterminate confirmational commitment fixed while data are collected and update using temporal credal conditionalization.
Although the indeterminacy in credal probability judgement will be fairly substantial at the outset, when the data are obtained, the extent of indeterminacy will be reduced.

Why does a shift to an indeterminate confirmational commitment constitute the appropriate change to make in the situation envisaged?

I am taking for granted (for the sake of simplicity) that there are no relevant differences in the states of full belief endorsed by George and Barbara. I also take for granted that when two agents or inquirers seeking to engage in a joint inquiry or enterprise wish to reason together, they should identify the common ground they share at the beginning of inquiry and start from there. One kind of argument for representing that shared agreement as the convex hull of the probabilities of George and Barbara, respectively, invites us to consider what should happen if George and Barbara are facing together a decision between horse lotteries in the sense of Anscombe and Aumann where George and Barbara agree in the utilities they assign to consequences (i.e., they agree in their preferences for roulette lotteries). Because of their differences in credal probability judgement, their preference rankings over all the possible horse lotteries definable as functions from the given states of nature and roulette lotteries over consequences will be different.

Consider, then, the quasi ordering of these horse lotteries obtainable from the Pareto agreements between George’s and Barbara’s preferences for the horse lotteries. This quasi ordering can be equivalently represented by the set of all weak orderings of the horse lotteries satisfying Anscombe–Aumann axioms and the constraints of the quasi-ordering (and not just George’s and Barbara’s). By hypothesis, the utility function for all consequences and roulette lotteries will be identical. Each ‘permissible’ weak ordering will generate a distinct probability function over the states. It can be shown that the set of all such probability functions is the convex hull of George’s and Barbara’s credal probability functions. If we think that Pareto unanimity represents shared agreement in the preferences of George and Barbara for the horse lotteries, the shared agreement will also be reflected in the convex set of credal probability functions just described.\(^7\)

When George and Barbara undertake a joint project, the shared agreements they reach at the outset constitute in effect the attitudes of the group agent of which they are constituent parts. That group agent can be criticized for failures of rationality and its attitudes identified just as in the case of a person. Not all groups are agents. I am inclined to
think that neither markets nor individuals participating in a prisoner's dilemma are. But in my judgement, if one wants to find good examples of agents carrying propositional attitudes that are not human, one should not look to automata or to animals, as is so often fashionable but to social agents.

I do not want to press the point too far here. But I do want to take note of one advantage of doing so. Not only can we draw conclusions about how group agents ought to behave from conditions of rationality imposed on human agents but the converse also holds. Not only groups, but humans like George and Barbara can be in suspense concerning credal probability and, hence, in indeterminate credal states. Indeed, in the example of group decision making we are considering, we are recommending that both George and Barbara revise their confirmational commitments so that they are in agreement. When they do so, the group they constitute will share with them the same credal state. Consequently, we have a somewhat better understanding of what it means for an individual person to embrace a position of doubt concerning credal probability than we may have had before.

Consider what it is to suspend judgement as to whether it will rain tomorrow. One must be in a state of full belief which has as a consequence that either it will or will not rain tomorrow but does not have as a consequence that rain will fall or that rain will not fall. That rain will fall and that rain will not fall are both serious possibilities. In general, they will both carry positive probabilities. They both have truth values. From the point of view of an inquiring agent prior to adding one of these conjectures to the state of full belief, adding either one of them incurs some risk of error.

Credal probability distributions do not represent truth value bearing claims. They cannot be assigned credal probabilities. They cannot be evaluated with respect to whether they are seriously possible or not. In these important respects, suspension of judgement between credal probabilities cannot be taken to resemble suspension of judgement between conjectures. Yet, I have suggested that being in a state of indeterminate credal probability judgement is analogous to being in a state of doubt or suspense concerning credal probability. Given the important respects in which such a state of doubt cannot resemble the doubts registered in a state of full belief, we need some explanation of the respects in which the doubts are analogous. The way in which decision making by appeal
to consensus as shared agreement can be elucidated in connection with group choice is useful in motivating the analogy.

Consensus as shared agreement concerning how to evaluate horse lotteries can with considerable plausibility be represented by appealing to the quasi-ordering generated Pareto unanimity. When this quasi-ordering is understood as representing shared agreements, it rules out as impermissible any weak ordering of the horse lotteries which is not a consistent extension of the pareto quasi-ordering. All weak orderings which satisfy the consensual quasi-ordering are recognized as permissible to use in evaluating the decision maker’s options. The permissible orderings are potential resolutions of the conflict between George’s and Barbara’s valuations. In consensual decision making, none of them are ruled out.

This view of doubt can be extended from the case of the group agent consisting of George and Barbara working in tandem. George and Barbara separately may shift their credal states and, hence, confirmational commitments to states which recognize all these orderings of the horse lotteries as permissible. As already indicated, such a state of indeterminate credal probability judgement (or doubt with respect to credal probability) will be representable by the convex hull of George’s initial credal state and Barbara’s.

But the idea need not be restricted to cases where persons are engaged in a joint endeavor and need to reach a decision by consensus. A Robinson Crusoe can also be in a state of doubt concerning credal probability judgement.

One kind of situation where this can happen arises where an inquirer begins in a state of full belief where he or she is convinced that some hypothesis \( H \) is false (so that \( H \) is incompatible with \( K \)). The inquirer needs to contract his or her state of full belief so that he or she is in suspense with respect to the truth of \( H \) and its negation (which may or may not be itself partitioned in a set of alternatives). For example, suppose that \( X \) is certain that a given brown mouse is the hybrid product of a purebred brown mouse and a purebred black mouse. The brown mouse is mated with a black mouse and has eight brown offspring. There is no inconsistency between this and \( X \)’s initial state of full belief. But prior to noting the eight brown offspring, \( X \) would have judged the chance of this result as extremely small, being equal to \( 1/64 \).

The result would be more understandable if the brown mouse were purebred, counter to the investigator’s opinion. The prospect of making
sense of the anomaly might constitute a good reason for the investigator contracting his state of full belief by removing the assumption that the brown mouse is purebred and shifting to suspense between this and the assumption that it is purebred.

The motivation for doing so is to give a hearing to the conjecture that the brown mouse is purebred. To give such a conjecture a hearing requires that one not beg the question against the conjecture. It requires also that one not beg the question in its favor. That is why contraction is required so that one moves to a ‘neutral’ position of suspense with respect to the truth values of the claims that the brown mouse is hybrid and that it is purebred.

Since the aim is to give a hearing to both alternatives from a neutral vantage point, we do not want the credal state relative to the contracted position to be skewed so that adding one of the alternatives via inductive expansion is warranted without further collection of data. We want the issue to be settled, if possible, by mating the brown mouse with a black mouse again and observing the colors of the offspring yielded. If the probability of one of the alternatives (say that the mouse is purebred) is very close to one prior to such data collection, the other hypothesis will be rejected via induction and the matter foreclosed before serious investigation can be begun. Consequently, to give the rival alternatives a fair hearing we want the prior credal state to be such that none of the alternatives can be rejected prior to such investigation.

How the prior credal state is to be determined via this principle depends upon the criterion for rejecting alternatives under scrutiny (and expanding the state of full belief by adding the information that one of the survivors is true). I have discussed criteria for deliberate or inductive expansion of belief states elsewhere (Levi, 1967, 1980, 1984, 1986, 1991) and shall not rehearse the matter here. One feature of those criteria is worth mentioning in this context. If among the permissible prior credal distributions there is at least one according to which none of the alternatives is rejected, then none of the alternatives is to be rejected according to the credal state and the criteria for inductive expansion.

Consequently, if one begins with a maximally indeterminate credal state recognizing all coherent distributions over the alternatives to be permissible, one will satisfy the requisite neutrality requirement. Unfortunately, if one adopts the maximally indeterminate credal state, the confirmational commitment embraced will not allow one to settle the issue concerning the alternatives on the basis of subsequent experimentation.
There can be no learning from experience. On the other hand, endorsing any specific distribution satisfying the neutrality requirement would appear arbitrary under normal circumstances. The sensible procedure would be to allow all and only distributions satisfying the neutrality requirement to be permissible.

To follow this procedure is tantamount to proceeding as if one were beginning from shared agreement with a hypothetical community of individuals each of which endorsed a different credal probability distribution over the alternatives meeting the neutrality requirement relative to the common criterion for justified inductive expansion. This fanciful analogy serves to remind us that giving a hearing to rival conjectures from a neutral vantage point is giving a hearing in a way which others who share the same standards of scientific rigor as the inquirer would be prepared to find convincing.

The two reasons for changing confirmational commitments just listed do not exhaust the possibilities. And additional elaboration is needed on the details of how and when these two kinds of reasons work. The aim here is to illustrate in outline two ways in which inquirers can break with confirmational tenacity and, hence, break with both temporal credal conditionalization and its inverse.

Thus, I share Suppes's scepticism concerning the strict Bayesian picture of a rational agent determining his current credal state by conditioning on an 'overall' probability (i.e., confirmational commitment) acquired early in life. Suppes rightly objects that this vision fails to give a complete account of how rational agents should change states of credal probability judgement. But even if we find an adequate account of rational change of full belief, confirmational tenacity remains open to question. We need an account of when and how confirmational commitments ought to be revised.

Department of Philosophy,
Columbia University,
New York, NY 10027, U.S.A.

NOTES

1 If we consider cases where the pertinent portion of the credal state is a continuous probability distribution over the real line, we might think of the space of serious possibilities as the 'support' of the distribution (i.e., smallest closed interval on the real line
carrying probability 1) with or without the end points. This does not determine whether the end points are serious possibilities. Even if we set this point to one side, the credal state fails to determine the set of serious possibilities. Points outside the support could be regarded as serious possibilities without violating any requirement on the coherence of probability judgement. And, in any case, the method for deriving a distinction between serious possibility and impossibility in this case cannot be generalized to other situations.

We do no better if we allow probability measures to take values in the nonstandard reals. We might distinguish between propositions counted as impossible and those that are possible but carrying O standard probability by assigning the latter but not the former infinitesimal credal probability. But to do this is simply another way of dividing the propositions into the seriously possible and impossible equivalent to appealing to the state of full belief. It remains true that one cannot derive the distinction from the credal state construed in terms of probability measures taking values in the standard reals. Appealing to nonstandard reals is conceding that one cannot regard the state of full belief as part of the credal state.

2 It is worth mentioning that both Jeffreys and Carnap were not rigid adherents to the conception of a standard confirmational commitment. Both of them realized the importance of modifying the confirmational commitment with a change in conceptual space or ‘language’. More interestingly, Carnap (1952) explored ways of choosing confirmational commitments (confirmation functions) relative to a fixed language on the basis of empirical considerations.

3 Changes in states of full belief, confirmational commitments and credal states will be discussed here under the unrealistic assumption that the agent is logically omniscient and emotionally stable. Otherwise, we would need to devise some way to contrast changes in these attitudinal states which are the product of inquiry and changes which are the product of therapy or technology which enhance the agent’s capacity to put two and two together. For further discussion of these kinds of changes, see Levi (1991, Ch. 2).

Hence, Jeffrey conditionalization is precluded in this kind of situation (Jeffrey, 1965). That is to say, it is precluded when it is claimed (as Jeffrey wants to claim) that the change in credal state is not representable as due to a change in the state of full belief expressible in a richer algebra or conceptual framework. Jeffrey conditionalization can occur if there is no change in state of full belief provided that there is a change in the confirmational commitment of the right sort.

4 In speaking of the total evidence or total knowledge requirement here, I do not mean to be focusing on whether the inquiring agent should be seeking new information to add to his current belief state. Ayer and many others following him suggested that the total available evidence requirement should be construed to recommend acquiring all the information via inquiry and observation which the inquirer is capable in some sense or other of doing. We have not been discussing this issue here. Following Seidenfeld (1979, Ch. 8), however, we emphasize that it is a different issue from the total evidence requirement.

5 It has sometimes been supposed that if an inquirer X is certain that A is true, he or she cannot contract his or her state of full belief by removing A. And this explains the neglect of inverse temporal; credal conditionalization. Often the only kind of expansion countenanced is expansion due to the making of observations. Changes in states of full
belief of other kinds are sometimes smuggled in as kinds of conceptual change. They are not conceptual changes in the sense deployed here which concerns changes in the space of potential states of full belief the inquirer is conceptually able or entitled to shift to. Changes in states of full belief are not changes in conceptual space. Those who think of some changes in potential states of full belief within a given conceptual framework as conceptual are introducing a distinction I do not understand well enough to discuss.  

7 When George and Barbara agree in their probability judgements but differ in their utilities, similar arguments can be used to support the idea that the consensus should be the convex hull of George’s and Barbara’s utility functions. Matters become more controversial when we consider cases where George and Barbara differ with respect to both probability and utility judgements. See Seidenfeld, Schervish and Kadane (1989), Levi (1990a and 1990b).

REFERENCES

From a broad philosophical perspective, Isaac Levi and I stand together on most of the large issues concerning changes in belief, especially as reflected in the title of his paper: 'Changing Probability Judgments'. We both have skepticism about various rigid orthodoxies for making such changes. No doubt the shared skepticism is something we both learned as graduate students from Ernest Nagel, although at somewhat different times.

In the framework of his paper, I also accept his exclusion of conceptual change as the focus of analysis, a topic that in other contexts I consider fundamental. Among various ideas that follow directly from our shared skepticism of overly precise formulations of how beliefs can be changed, I emphasize the following. We are both sympathetic to the use of upper and lower probabilities as, in many cases, approximations that are more realistic than the kind of unique probability distributions required by pure Bayesians. I especially agree with the two types of occasions on which it is natural to change what Isaac calls 'confirmational commitments'. Both cases he stresses are of great importance. The first is changing, in my terms, one's partial probabilistic beliefs in order to accommodate for any of a variety of reasons the opinion of someone else. The important thing is that we do make such adjustments to take account of the opinion of others and it is one of the ironies of the Bayesian viewpoint that so little technical consideration has been given to the detailed way in which such adjustments in views might be made. Of course there is the standard answer that experiments that are jointly conceived can be performed, but that is not really the point. We often change our own opinions, i.e., partial beliefs, on the basis of hearing someone else's opinion, not on the basis of hearing about detailed experiments. The psychologically descriptive or normative account of such changes is still in its infancy, but Levi and I very much agree on its importance.

The second and related view is to apply the same concept to Robinson Crusoe, i.e., to a single individual. We can naturally and easily engage in a suspension of belief while investigating alternative hypotheses.
In the process of such investigation and as a conclusion as well, we modify at all stages our partial beliefs. I like Levi's detailed genetic example toward the end of the paper, and I also very much agree with his characterization of this class of occasions for changing beliefs.

I now want to turn to some of our differences. It is important, for those who are not familiar with all of the details of the discussions on many different kinds of points, to realize that a sharp and exact statement of differences is essential for those who agree in broad perspective, in order to refine the conception of how beliefs are changed. I have many points of disagreement with Isaac but none of my views is held dogmatically. I set them forth because I think they are criticisms that should be made at this stage even if some of them may move the discussion in the wrong direction. I list my points of difference, some of them certainly minor.

**Set of Credal States.** I am skeptical of and find unrealistic his characterization of a credal state as a convex set of probability distributions, each of which is consistent with the person's state of full belief $K$ at the time. Such sets are quite unmanageable objects. It seems unrealistic simply to adopt them without a full analysis of the computational implications of doing so. One approach is to give qualitative axioms, which may be the ones actually at work in an individual's formation of his credal state, and these axioms lead to the existence of such a convex set of probability functions. The axioms themselves would need to be highly constructive in character.

**Closure of Credal States.** The same problem arises a little later in requiring that the credal state be closed under consequence. Again a computationally unrealistic idea for actual believers and even for computers, given that in general for languages and theories of any complexity, closure will be at best recursively enumerable not recursive. Closure of beliefs under consequence, and even more unrealistically, closure of knowledge under consequence is a topic that has a long history of discussion in philosophy and represents for me a quite unrealistic idealization for any rational believer this side of the divine.

**Problem of Certainty of Belief.** I am also enough of a Bayesian, at least of my own stripe, to be skeptical of probability 1 statements. One
might say ‘what about logical truths?’ Again, on the other side of the divide, all logical truths are known as well as everything else, but logical truths do not come with linguistic labels. The computational problem of deciding if something is a logical truth may be far beyond me, so that even logical truths cannot be assigned probability 1. I am sufficiently skeptical of my own errors that even for quite simple statements I will reserve a very small bit of positive probability to express skepticism about their truth, just in case I have misread a negative sign or another sentential connective. The sharp split that Levi makes between full belief and the credal state is not one that I would accept.

Space of Potential Cognitive States. Now I come back to a point that occurred at the beginning but I thought would be best not to start with, and that is the fixing of a space of potential cognitive states as the apparatus with which to work. This great drive for extensional precision of statement of structure is one that is hard to resist and one that I myself have not been able to resist in the past. In this case I share de Finetti’s skepticism of having a fixed probability space of possible outcomes, which in the present context translates into Levi’s space of potential cognitive states. The world does not slice up so nicely into possible cognitive states or possible outcomes. It is more complicated, subtle and intriguing and, in spite of the widespread use of this methodology in many parts of philosophy and science, skepticism is in order. In fact, this calls for one other comment. The widespread use of random variables in statistics is, in fact, a way of avoiding commitment to a fixed sample space. What is required technically in that standard use is just Kolmogorov’s theorem on the existence of a common probability space, i.e., a common space of possible outcomes for a family of random variables whenever every finite set of the random variables has a joint distribution. But note, it is existence not uniqueness of this probability space that is needed for the standard technical framework. Uniqueness is essentially never examined and of no interest, which is in itself a good argument not to begin with some fixed space of potential outcomes.

Skepticism about Supporting Probabilities. I now want to turn to the aspects of my current views that are truly more radical than Levi’s. I am skeptical of even having in many situations a credal state reflecting upper and lower probabilities that are representable in terms of a convex set of
definite probability distributions. I have mentioned the computational aspect and I now want to consider the specific kinds of examples that lead to the existence of such a convex set being itself incompatible with the beliefs held. Consider a physician who is asked to remark on the correlation of various symptoms for various diseases. It is conjecture, but I am sure not difficult to substantiate empirically, that many of the beliefs of physicians in these matters are not coherent, i.e., they cannot be represented by a probability distribution. Consider three symptoms \( s_1, s_2, \) and \( s_3 \). The correlation between \( s_1 \) and \( s_2 \) is \(-1/2\) and similarly for the other two pairs. It is easy to show that there cannot be a joint probability distribution of the three symptoms even though there is a proper pairwise distribution for each pair. It is one of the fantasies of the Bayesian view of the world, as I have emphasized on numerous occasions, that people can hold realistic prior probabilities about a large number of concepts or variables at the same time. When the number is even as much as four or five, we are lucky to be able to have some sense of the expectation of each individual variable, and the pairwise correlations. Once we have more than three or four, the computational task of deciding whether pairwise correlations are coherent – not to speak of the distributions of triples, quadruples, etc. – is a genuinely complicated computational task. It seems to me important, therefore, in thinking realistically about changing beliefs, that we need to have clarity about what kind of states we start with.

The states we start with, I would say, are in general not representable even by a convex family of probability distributions, for the reasons just stated. In some cases, as reflected in correlational data of physicians’ beliefs, we may be able to represent the beliefs by a nonmonotonic upper probability of the kind studied by Zanotti and me (Suppes and Zanotti, 1991) for application to the Bell inequalities in quantum mechanics, but now applied without formal change as a nonmonotonic upper probability for incoherent beliefs. Given this conception of our beliefs, the puzzles increase. How do we change our beliefs given that we start from an incoherent state and in many cases end up in an incoherent state? In the case of a rigorous framework of inference to be published in the appropriate scientific journal or the like, can we proceed in a realistic way from an initially incoherent set of beliefs? The answer to this last question is I think clearly yes, but the answer to the first is more puzzling as to how we should think about a continual state of incoherent belief. The first thing we need, and this is along the lines of Levi’s very last
remark in his paper, is much more thinking both about how we can represent beliefs of rational but computationally limited entities, and how can such rational but computationally limited entities change their beliefs? We are, in my own view, only at the beginning of the first pages of the first chapter of this complex and subtle story.

REFERENCE