Imprecise Probabilities in Quantum Mechanics

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Motivation

- In his entry on "Quantum Logic and Probability Theory" in the *Stanford Encyclopedia of Philosophy*, Alexander Wilce writes that "it is uncontroversial (though remarkable) that the formal apparatus of quantum mechanics reduces neatly to a generalization of classical probability in which the role played by a Boolean algebra of events in the latter is taken over by the 'quantum logic' of projection operators on a Hilbert space."
- For a long time, Patrick Suppes has opposed this view. Instead of changing the logic and moving from a Boolean algebra to a non-Boolean algebra, one can also 'save the phenomena' by weakening the axioms of probability theory and working instead with upper and lower probabilities or provisions (Walley 1991).

Motivation (Cont'd)

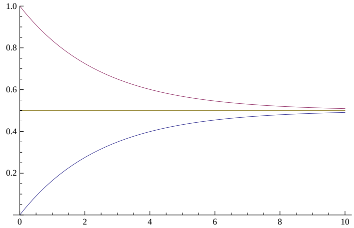
- However, it is fair to say that despite Suppes' efforts upper and lower probabilities are not particularly popular in physics as well as in the foundations of physics, at least so far.
- Instead, quantum logics is booming (again), especially since quantum computation became a hot topic.
- In this short presentation I would like to give one more reason for the use of upper and lower probabilities in quantum mechanics and the research program that they inspire.
- To do so, I have to be sketchy and will focus on the "big picture".

Overview

1. Imprecise Probabilities
2. Quantum Mechanics and Its Puzzles
3. Imprecise Probabilities in Quantum Mechanics
4. Open Questions

Imprecise Probabilities

- How to represent ignorance about a probability value?
- Introduce a (super-additive) lower probability measure P_* and a (sub-additive) upper probability measure P^*.
- Idea: If more and more evidence is collected, e.g. in a coin tossing experiment, the lower and upper probability approximate the probability distribution.
- Interpretation: Minimal/maximal betting odds (Walley 1991).



What has this to do with quantum mechanics?

The CHSH Inequality

- Consider four random variables A, A', B, B' that can take the values ± 1 and define

$$\mathcal{F} := |E(AB) + E(AB') + E(A'B) - E(A'B')|.$$

- The expectation value $E(AB)$ measures the correlation between A and B and can take the values ± 1. It can be observed.
- Generalizing a celebrated result from Bell, the physicists Clauser, Horn, Shimony and Holt (effectively) showed that:

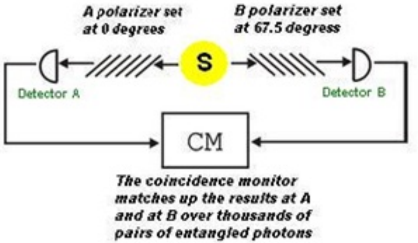
  If there is a joint probability distribution $p(A, A', B, B')$, then $\mathcal{F} \leq 2$ ("CHSH inequality").

- NB: The correlations do not uniquely fix the joint distribution.

Quantum mechanics violates the CHSH inequality.

Violation of the CHSH Inequality

- Consider the quantum state $|\mathrm{EPR}>= \frac{1}{\sqrt{2}}(|10> - |01>)$.
- Here $|0>$ and $|1>$ represent the states of the two subsystems $\mathcal{A}$ and $\mathcal{B}$, e.g. certain photon polarizations.
- Then measurement angles α, α' (at $\mathcal{A}$) and β, β' (at $\mathcal{B}$) can be found such that $\mathcal{F} = 2\sqrt{2} > 2$. Hence, CHSH is violated.
- Note: $2\sqrt{2}$ is the maximal violation of the CHSH inequality.



The CHSH Inequalities for Atoms

- Related experiments can be made with two level atoms which are trapped, e.g. in a cavity. Here $|0>$ and $|1>$ represent the states of a single 2-level atom being in the ground state or the excited state, respectively.
- One gets a maximal violation of the CHSH inequality (i.e. $2\sqrt{2}$), if one sets

$$A := X_1 \quad , \quad A' := Z_1$$
$$B := X_2 + Z_2 \quad , \quad B' := X_2 - Z_2$$

- Here X_1 corresponds to the Pauli matrix σ_x applied to the state of subsystem 1. Z_1, X_2 etc. are defined accordingly.

Decoherence

- If not stabilized, the EPR state will decay as time progresses, and the pure state becomes the mixture

$$\rho(\tau) = e^{-\tau}\,\rho(0) + (1 - e^{-\tau})\,|00><00|,$$

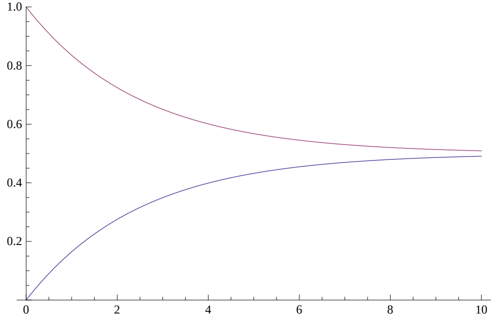
with the initial state $\rho(0) = |\mathrm{EPR}><\mathrm{EPR}|$. The dimensionless parameter τ is the time in units of the decay constant B.

- From the CHSH inequality we conclude that there is a joint probability distribution (and henceforth a classical description of the correlations) if $\tau > .217$.
- But how should we account for the quantum correlations before that time? What explains the correlations?

Imprecise Probabilities in QM

- There is always a (typically non-unique) upper probability distribution that "converges" to the joint distribution once the CHSH inequalities are fulfilled.
- Often, but not always, there is also a lower probability distribution P_* with $P_*(A) \leq P^*(A)$.
- N.B.: The lower and the upper probability distribution are, in the quantum mechanical case, typically not related via $P_*(A) = 1 - P^*(\overline{A})$ due to a violation of the monotonicity condition.

The Situation in QM



Upshot

- Decoherence fits well into the picture of upper and lower probability distributions as we can precisely understand how upper and lower probability distributions evolve over time and eventually degenerate into joint probability distribution.
- This transition from the quantum to the classical is harder to understand from the point of view of quantum logics. What does it mean that a non-Boolean algebra transforms into a Boolean algebra once the CHSH inequalities are satisfied? How can this process be modeled? – I do not want to say that this is not possible, but it seems much harder.

Open Questions

1. How do our results generalize to other states and systems?
2. What is the interpretation of upper and lower probabilities in quantum mechanics? (Walley's betting interpretation does not work as there is no joint probability distribution.)
3. How are they related to negative probabilities (for which the decoherence story can be told as well)?
4. Are there upper and lower probabilities if $2\sqrt{2} < \mathcal{F} \leq 4$? What distinguishes the quantum domain from that domain in terms of upper and lower probabilities (Popescu 2011)?
5. Can one say more about the analogy between learning and decoherence?

...enough to do for the years to come...