Interpreting Quantified Noun Phrases in Doubly Extended Relation Algebras

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1970s papers developing an algebraic semantics for natural languages:

- Suppes, P. (1976) Elimination of quantifiers in the semantics of natural language by use of extended relation algebras. *Revue Internationale de Philosophie* **117-118**, 243-259.
- Suppes, P. and Macken, E. (1978) Steps toward a variable-free semantics of attributive adjectives, possessives, and intensifying adverbs. In K. Nelson (ed.) *Children's Language*. Gardner Press. pp. 81-115.
- Suppes, P. (1979) Variable-free semantics for negations with prosodic variation. In E. Saarinen et al. (eds.) *Essays in Honour of Jaakko Hintikka*. Reidel. pp. 49-59.
- Suppes, P. (1982) Variable-free semantics with remarks on procedural extensions. In T.W. Simon and R.J. Scholes (eds.) *Language, Mind, and Brain*. Erlbaum. pp. 21-34.

Reaction against Richard Montague's *Proper Treatment of Quantification in Ordinary English*, as a semantics that

- inevitably fits the syntax of natural languages awkwardly,
- requires the use of individual variables where they do not actually occur, and
- needs a full Zermelo-like hierarchy of sets for the model structures of ordinary sentences.

Suppes papers argue that algebraic semantics

- has obvious advantages in efficiency of computing meanings and avoids artificially escalating semantic types

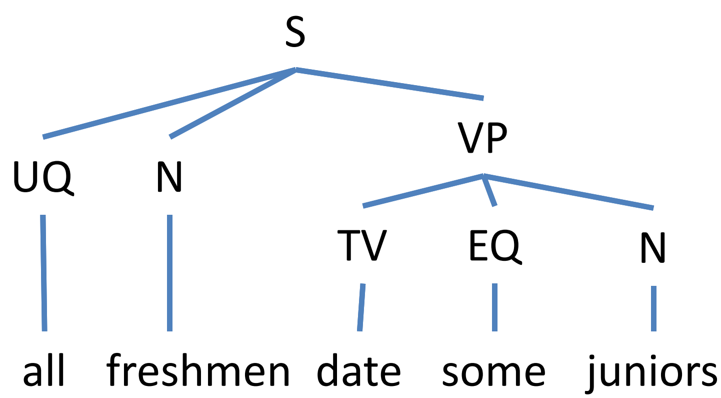
but also

- is at odds with established thinking about the syntax of natural languages.[1]

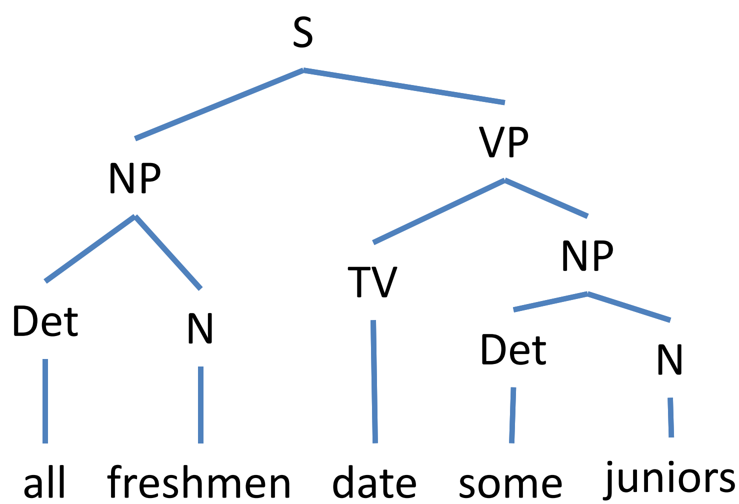
[1]Expressions like "all freshmen" and "some juniors" that can each be substituted for any occurrence of another without affecting the well-formedness of sentences constitute phrases of the syntactic type NP (or noun phrase).

Why did Pat conclude that the phrase structure of the sentence "all freshmen date some juniors" must be (1) instead of (2)?

(1)



(2)



A. He aimed to semantically interpret English in an extended relational algebra.

Def. Given a set D, a subset $\mathfrak{R}$ of the union of Pow(D) with Pow(D^2) is an *extended relational algebra* (over D) iff $\mathfrak{R}$ is closed under complement and union, $\mathfrak{R} \cap$ Pow(D^2) is closed under inverse and composition, and when $R \in$ Pow(D^2) and $S \in$ Pow(D), the image of S under R (viz. $R`S = \{a \in D : \exists b \in S\ R(a,b)\}$) $\in \mathfrak{R}$.

B. He argued that such an algebra does not provide sufficient denotations to correctly interpret every quantified noun phrase that contains a noun denoting a subset of D.

Quantified NPs Can't Denote Individuals or Sets Thereof!

- Truth conditions

(1) All As are Bs is true iff $[\mathbf{A}] \subseteq [\mathbf{B}]$

(2) Some As are Bs is true iff $[\mathbf{A}] \cap [\mathbf{B}] \neq \varnothing$

- Sentence meaning is compositional in the meanings of the subject NP and predicate VP

(3) T(**[**all As**]**, **[**B**]**) iff $[\mathbf{A}] \subseteq [\mathbf{B}]$

(4) T(**[**some As**]**,**[**B**]**) iff $[\mathbf{A}] \cap [\mathbf{B}] \neq \varnothing$

- A simple combinatorial argument shows

Theorem. These conditions require at least $2^{|D|+1} - |D|$ distinct denotations for **[**all As**]** and **[**some As**]** when **[**A**]** and **[**B**]** can be arbitrary subsets of D.

Proof: Set f_{all}(**[**A**]**) = **[**all As**]** and f_{some}(**[**A**]**) = **[**some As**]** for all As and **[**A**]** $\subseteq$ D.

Lemma 1. (3) implies f_{all} is 1–1; (4) implies f_{some} is 1–1.

Lemma 2. (3) and (4) jointly imply that if f_{all}(**[**A_1**]**) = f_{some}(**[**A_2**]**), then **[**A_1**]** = **[**A_2**]** = $\{a\}$ for some a $\in$ D.

Extending the Relational Algebra Further

Given D

1. *n*-ary first-order relations: Pow(D^n)

2. *m*-ary second-order relations over *n*-ary relations: Pow([Pow(D^n)]^m)

[Note: For *1*, just *n* = 0,1,2,3 are needed.

For *2*, just *n* = 1 and *m* = 1,2 are needed.]

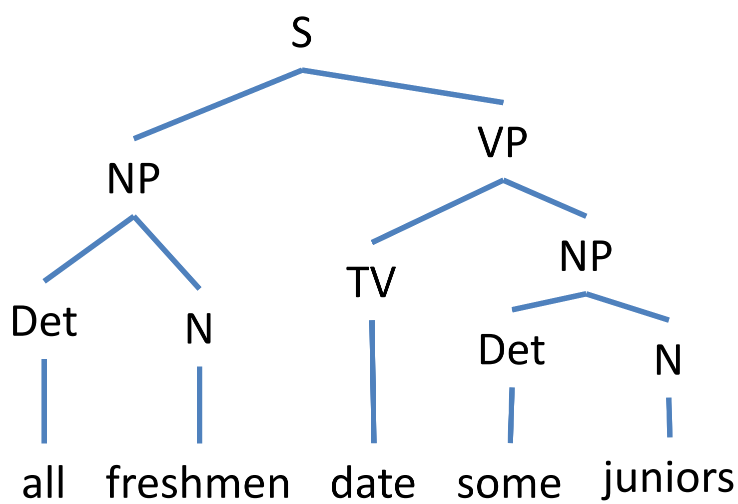
- Add the image operation R‘‘S for binary relations R and appropriate arguments S: R‘‘S = {T : R(T,S)}.
- Add the following operation taking a unary second-order Q and an *n*-ary first-order R to the (*n*–1)-ary first-order relation Q‘‘‘ R defined as follows.

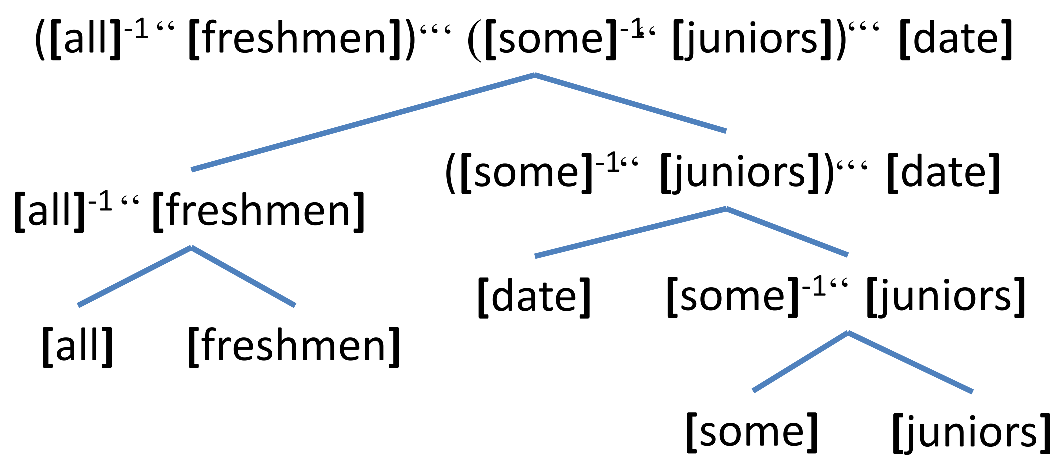
  Q‘‘‘ R(a_1,…,a_{n-1}) iff Q({a : R(a_1,…,a_{n-1},a)})

Semantic Interpretations

- Dets like “all”, “most”, “some”, “no”, etc. as binary relations between subsets of D — that is, in Pow([Pow(D^1)]^2)
- Ns like “freshmen” and “juniors” as unary first-order relations (members of Pow(D^1), i.e. subsets of D)
- The subcategories of Vs as *n*-ary first-order relations — viz. IVs like “stood” in Pow(D^1), TVs like “date” in Pow(D^2), and DTVs like “gave” in Pow(D^3).
- Semantic Composition rules:
  - **[**NP**]** = **[**Det**]**$^{-1}$ ‘‘ **[**N**]**
  - **[**S**] = [**NP**]**‘‘‘ **[**VP**]**
  - **[**VP**]** = **[**IV**]**
  - **[**VP**]** = **[**NP**]**‘‘‘ **[**TV**]**
  - **[**VP**]** = **[**NP_1**]** ‘‘‘ **[**NP_2**]**‘‘‘ **[**DTV**]** (Note: associate to the right)

Semantic Trees





Summing Up

Thus extending relational algebras slightly further enables one to have an algebraic semantics of natural languages which preserves Frege's insight that quantifiers like "everything", and even including "all freshmen" and "most juniors", are second-order concepts – unary second-order relations on unary relations on the domain. Moreover, one can formally capture Aristotle's implicit insight that determiners like "all", "no", "some", etc. are relations between sets – binary second-order relations on unary relations on the domain.

The postulated algebraic semantic values are efficient to compute, respect the linguistically motivated syntactic structure of sentences, and neither escalate semantic types nor necessitate artificially inserting variables where actually none exist. They also interpret sentences containing 'non-syllogistic' determiners like "most".

This is an algebraic semantics with the properties Pat desired, including "snugly fitting the syntax of natural languages as subtly as could be desired."