Indirect (i.e. Derived) Measurement and Evidence

George E. Smith, Tufts University

In the 90th year of Patrick Suppes

"Indirect measurement": ***Determining the values of a quantity A by measuring a quantity B systematically linked to A according to a theoretical principle***

"Evidence": ***Under what circumstances do well-behaved results in the values for A obtained thus indirectly provide evidence in support of the theoretical principle linking A to B that licenses the measurement?***

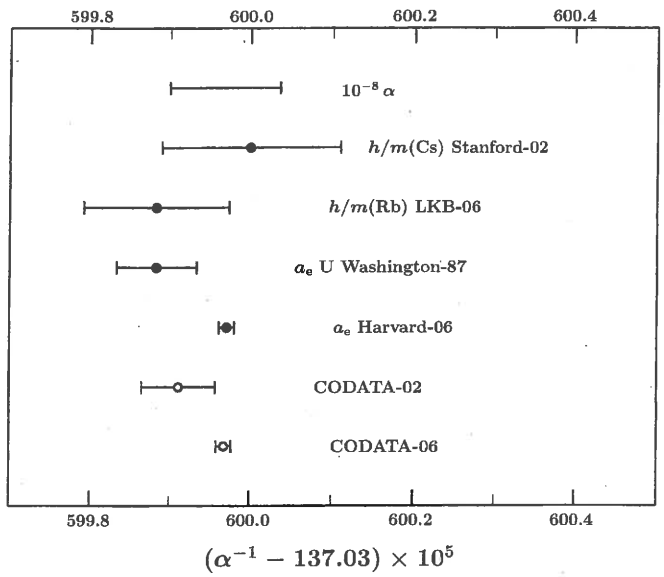


FIG. 4 Values of the fine-structure constant α with $u_r < 10^{-8}$ implied by the input data in Table XXX, in order of decreasing uncertainty from top to bottom. (See Table XXXIV.)

"The good agreement of the highly accurate values of α inferred from $h/m(^{133}\text{Cs})$ and $h/m(^{87}\text{Rb})$, which are only weakly dependent on QED theory, with the values of α inferred from a_e, muonium transition frequencies, and H and D transition frequencies, provide support for the QED theory of a_e as well as the bound-state of muonium and H and D. In particular, the weighted mean of the two values of α inferred from $h/m(^{133}\text{Cs})$ and $h/m(^{87}\text{Rb})$,

$$\alpha^{-1} = 137.035\,999\,34(69)\ [5.0\times10^{-9}],$$

and the weighted mean of the two values α inferred from the two experimental values of a_e,

$$\alpha^{-1} = 137.035\,999\,680(94)\ [6.9\times10^{-10}],$$

differ by only $0.5u_{diff}$, with $u_{diff} = 5.1\times10^{-9}$. This is a truly impressive confirmation of QED theory."

2006 CODATA, p. 86

Newton's First Two Laws of Motion

***Law 1*: Every body perseveres in its state of being at rest or of moving uniformly straight forward except insofar as it is compelled to change its state by forces impressed.**

***Contraposing*: Any departure of a body from rest or from uniform motion along a straight line requires an (unbalanced) impressed force acting on the body.**

***Law 2*: A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.**

***Contraposing*: Any motive force compelling a body to depart from its state of being at rest or of moving uniformly straight forward acts along the line of and is proportional to that change of state.**

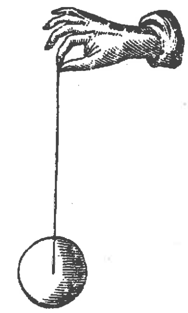
***On Newton's interpretation*: In any continuous change of motion of a body, the magnitude of the change at any time is proportional to the limiting value as δt approaches 0 of δs and the mass of the body directly and the square of δt inversely, where δs is the line segment extending from where the body is to where it would have been in the absence of any change in state.**

***The licensed inference*: FROM the limiting value of $m\times\delta s/(\delta t)^2$ at any time TO the magnitude and direction of the impressed motive force compelling the change at that time (*without regard to any question of the mechanism effecting the force*)**

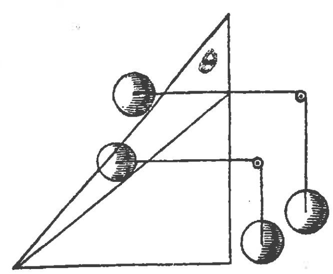
***For example*: The mean value of the component of the solar force on the Moon acting toward the Earth is 1/178 29/40 times the mean value of the Earth's force on the Moon.**

Some Pertinent Principles from Statics

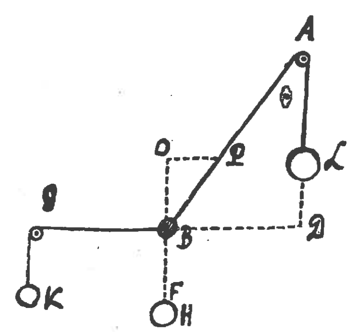
The tension in a vertically suspended string produced by a body hanging from it varies as the density and volume of the body and the strength of the tendency bodies at the location in question have to descend.



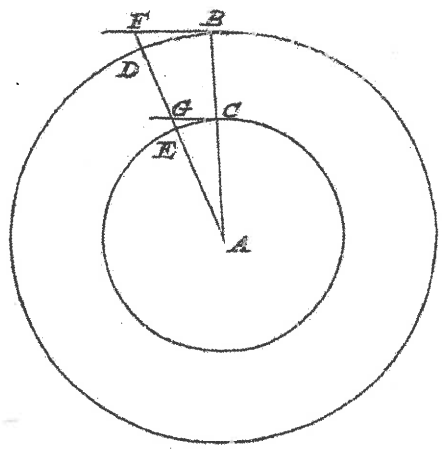
The tension in a string retaining two bodies in static equilibrium, one vertically and the other on an inclined plane, varies as the cosine of the angle of the plane θ and the tension in a vertically suspended string that would be produced were the body on the inclined plane hanging from it.



The tension in a string BK required to maintain a body in equilibrium at an angle θ varies as the tangent of that angle and the tension in string BH when that body is hanging vertically at its end.



Huygens's Centrifugal Tension in a String



The tension in a string retaining a body in uniform circular motion is proportional to the product of the density and volume of the body and the limiting value as G $\Rightarrow$ C as EG/δt^2.

But, by Euclid III 36, in that limit

$$\mathrm{EG}/(\delta t)^2 \propto [\mathrm{GC}^2/\mathrm{AC}]/\delta t^2 = v^2/\mathrm{r} = 4\pi^2 r/P^2$$

where the period *P* is the time required to complete one circuit.

Huygens's theory of *centrifugal force* thus theoretically linked Newton's change of motion to a static force, namely the tension in a string retaining a body in circular motion.

HUYGENS'S PARABOLOIDAL CONICAL PENDULUM CLOCK MEASUREMENTS OF SURFACE GRAVITY (1673)

Distance of fall in 1st second

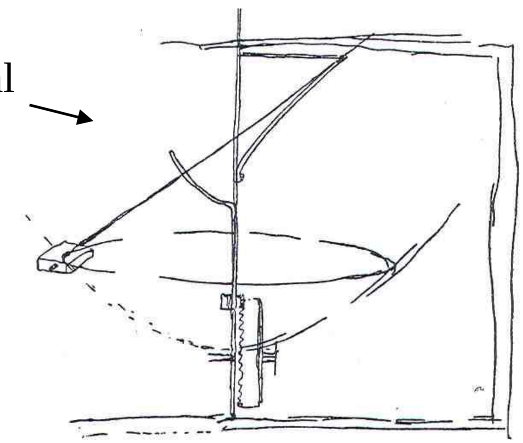
$= \pi^2 \cdot$latus rectum$/P^2$

Latus rectum of a one-half sec paraboloidal-conical-pendulum clock: 4 inches 7.1 lines

Distance of fall in 1st second:

15 Paris feet 1.1 inches

Original sketch



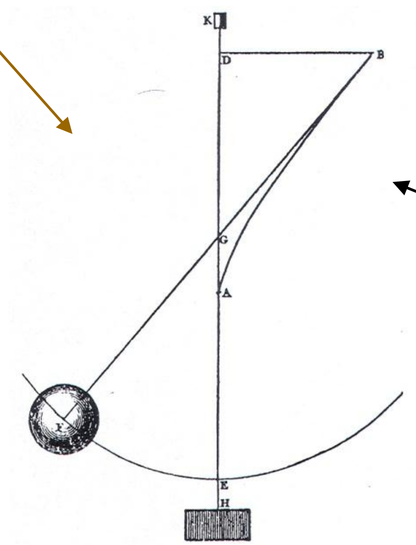


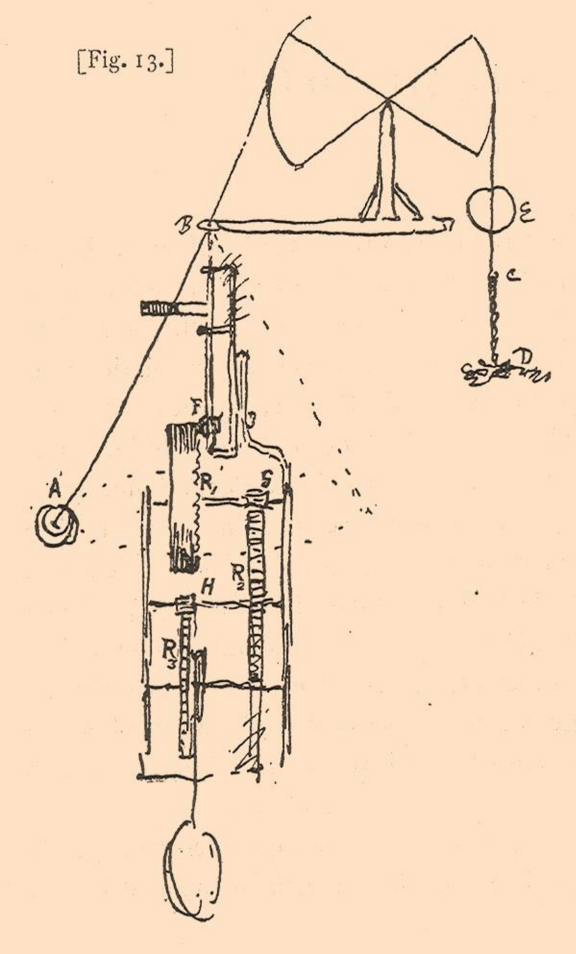
Diagram in *Horologium Oscllatorium*

HUYGENS'S CONSTANT-HEIGHT CONICAL PENDULUM MEASUREMENT OF THE STRENGTH OF SURFACE GRAVITY (1659)

Distance of fall in 1st second

$= 2\pi^2 \cdot$height of pendulum$/P^2$

Height of a three-quarter sec constant-height conical-pendulum clock: 5 inches 1.9 lines



Distance of fall in 1st second:

15 Paris feet 1.1 inches

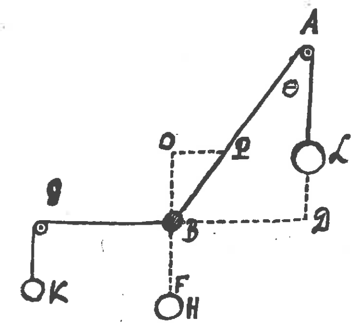
Evidence for Newton's First Two Laws in 1687

<u>On the one hand</u>, Huygens's conical pendulum measurements of surface gravity had shown that

$$\lim \delta s/(\delta t)^2 \propto \textit{constant} \times \text{tangent } \theta$$

where the constant involves an empirically invariant relationship (to within the 4th significant figure) between a dimension of the conical pendulum and the period squared:

$$\textit{constant} = (2\pi^2 h)/P^2 = (\pi^2 L)/P^2$$



<u>On the other hand</u>, statics (Stevin) had shown that static equilibrium requires

tension in BK = *vertical-tension* in BH × tan θ

But Huygens had also devised another indirect measure of the tendency to descend vertically, using cycloidal and small-arc circular pendulums, that had become standard and that:

1. Did not, as such, presuppose either of Newton's first two laws, but presupposed instead Galileo's uniform vertical acceleration and pathwise independence of velocity acquired in descent

2. Gave the same value 15 Paris ft 1.1 inch value as the conical pendulum measures, i.e. the above constant

"What has been demonstrated concerning the times of oscillating pendulums depends on the same first two laws and first two corollaries and is supported by daily experience with clocks."

Patrick's Gestured Question

What evidence, if any, do well-behaved results obtained in an indirect measurement provide in support of the theoretical principle licensing that measure when there are no complementary indirect measures of the same quantity licensed by different theoretical principles?

For example, what evidence, if any, would the well-behaved results obtained in the conical pendulum measurements of *distance of fall in the 1st second* have provided in support Newton's first two laws of motion as of 1687 if there had been no other way of measuring that quantity with any precision?

Evidence for Newton's First Two Laws in 1687

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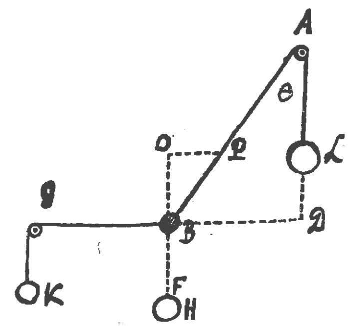
***tension* in BK = *vertical-tension* in BH × tan θ**



But if the vertical tension in BH is constant and repeatable at any one location, the correlation implied by the similarity of form of the above expressions still allows the quantity *mass* × $\lim \delta s/(\delta t)^2$ to be a way of assigning definite values to *force*, with the latter in full accord with established measures of static forces.

***The question*: Are there any systematic differences between static forces and the quantity *mass* × $\lim \delta s/(\delta t)^2$ that limit the range and precision of the latter as an indirect measure of *force*?**

Evidence for Newton's First Two Laws in 1687

<u>On the one hand</u>, Huygens's conical pendulum measurements of surface gravity had shown that

$$\lim \delta s/(\delta t)^2 \propto \textit{constant} \times \text{tangent } \theta$$

where the constant involves an empirically invariant relationship (to within the 4th significant figure) between a dimension of the conical pendulum and the period squared:

$$\textit{constant} = (2\pi^2 h)/P^2 = (\pi^2 L)/P^2$$

But the well-behaved results from conical pendulums show that the quantity $\underline{\textit{mass} \times \lim \delta s/(\delta t)^2}$ is measuring *something*, and that by itself amounts to a successful test of the first two laws.

For, if the conical pendulum measurements had not yielded a value invariant within their bounds of precision -- as predicted – they would have been evidence against the first two laws.

***The question then*: What does the quantity $\underline{\textit{mass} \times \lim \delta s/ (\delta t)^2}$ really measure, and how, if at all, is that related to established ways of measuring static forces?**

Definition 1**: Quantity of matter is a measure (*****mensura*****) of matter that arises from its density and volume jointly.**

Definition 2**: Quantity of motion is a measure of motion that arises from the velocity and the quantity of matter jointly.**

Definition 8**: The motive quantity of centripetal force is the measure of this force that is proportional to the motion it generates in a given time.**

Foundations of Measurement**, volume 1,**
Additive and Polynomial Representations

David H. Krantz, R. Duncan Luce,
Patrick Suppes, and Amos Tversky

1971