Roberta Ferrario

WHO CARES ABOUT AXIOMATIZATION?
REPRESENTATION, INVARIANCE, AND
FORMAL ONTOLOGIES

1. INTRODUCTION

The philosophy of science of Patrick Suppes is centered on two important notions that are part of the title of his recent book (Suppes [2002]): Representation and Invariance. Representation is important because when we embrace a theory we implicitly choose a way to represent the phenomenon we are studying. Invariance is important because, since invariants are the only things that are constant in a theory, in a way they give the “objective” meaning of that theory.

Every scientific theory gives a representation of a class of structures and studies the invariant properties holding in that class of structures. In Suppes’ view, the best way to define this class of structures is via axiomatization. This is because a class of structures is given by a definition, and this same definition establishes which are the properties that a single structure must possess in order to belong to the class. These properties correspond to the axioms of a logical theory.

In Suppes’ view, the best way to characterize a scientific structure is by giving a representation theorem for its models and singling out

the invariants in the structure.

Thus, we can say that the philosophy of science of Patrick Suppes consists in the application of the axiomatic method to scientific disciplines.

What I want to argue in this paper is that this application of the axiomatic method is also at the basis of a new approach that is being increasingly applied to the study of computer science and information systems, namely the approach of formal ontologies.

The main task of an ontology is that of making explicit the conceptual structure underlying a certain domain. By "making explicit the conceptual structure" we mean singling out the most basic entities populating the domain and writing axioms expressing the main properties of these primitives and the relations holding among them.

So, in both cases, the axiomatization is the main tool used to characterize the object of inquiry, being this object scientific theories (in Suppes' approach), or information systems¹ (for formal ontologies).

In the following section I will present the view of Patrick Suppes on the philosophy of science and the axiomatic method, in section 3 I will survey the theoretical issues underlying the work that is being done in formal ontologies and in section 4 I will draw a comparison of these two approaches and explore similarities and differences between them.

2. PATRICK SUPPES: FORMAL METHODS IN THE PHILOSOPHY OF SCIENCE

2.1 The First Pillar: Representation

The recent Suppes' book, Representation and Invariance of Scientific Structures (Suppes [2002]), collects many of the ideas characterizing the last forty years or so of his philosophical enterprise. The main purpose of this book is that of showing how the enormous building of his philosophy of science is founded on these two very

¹ An information system is a system that collects and stores data; an example of information system is a database.
important concepts, representation and invariance, which I have called in this paper “the pillars” of his approach. Let me start with a brief analysis of the first pillar, namely representation.

In order to explain what he means when he talks about representation of scientific theories, Suppes begins with a common-sense characterization of the concept. The first things that come to mind when people ordinarily think about representation are images and reproductions of other sorts or, in a slightly more sophisticated fashion, models. In all these cases, the image, reproduction, or model are there “on behalf of” the thing that we want to represent.

Another very important and intuitive feature of representations is that they are always partial with respect to the things they represent, in the sense that they necessarily display some features of these things and disregard others. The only thing that can represent exactly an object in all its details is the object itself. This feature has a straightforward advantage: by focusing only on some particular aspects that are evaluated as relevant by the person who is performing the representation task, it helps in better understanding the object of inquiry. Why it is so is not too difficult to explain: the process of representation can also be seen, at least in part, as a process of reduction; a reduction from the unknown to the known: we give a representation of something that is in part unknown to us (and that we want to represent in order to analyze it), by means of the aspects of this something that we already know. When we accomplish this task of reduction in a satisfactory way, we can say that we have a good representation. A good representation, in this way, can shed new light on the object of our analysis.

One of Suppes’ favorite examples (Suppes [1988]) of successful reduction is due to Descartes, who also gives, in another domain, an example of bad reduction. In the former case, what is at issue is the reduction of geometry to algebra, where Descartes succeeded in representing geometrical problems in algebraic terms, thus introducing measurements in a discipline that at the time (and since Euclid, actually) was mainly based on ratios. Descartes’ reduction of geometry to algebra is very precise and formal in character. In the second case, we are talking about the reduction of matter to extension, that was
doomed to failure, being driven by a sort of "speculative mood", and can hardly be considered scientific. This focus on reduction brings us closer to Suppes' analysis of representation in scientific theories, and thus to the first pillar of his philosophy of science.

In Suppes' view, the best way to accomplish the task of philosophy of science, i.e. to study and analyze the structure of scientific theories, is by means of the analysis of their models (Suppes [1988]). And this is exactly where representation comes into play, because, according to Suppes, in order to study the models, we have to give representation theorems for them.

Here is what Suppes means by giving a representation theorem for the models of a scientific theory (Suppes [1988], pp. 259-260):

Perhaps the best and strongest characterization of the models of a theory is expressed in terms of a significant representation theorem. By a representation theorem for a theory the following is meant. A certain class of models of a theory distinguished for some intuitively clear conceptual reason is shown to exemplify within isomorphism every model of the theory. More precisely, let $M$ be the set of all models of a theory, and let $B$ be some distinguished subset of $M$. A representation theorem for $M$ with respect to $B$ would consist of the assertion that given any model $M$ in $M$ there exists a model in $B$ isomorphic to $M$.

So, in a way, the representation theorem allows us to concentrate our attention on those models that, containing some features that are particularly relevant for the theory, can be good representatives for that same theory.

It can be noticed that also the element of reduction is present in this characterization of the representation process: the representation theorem reduces all the models of a theory to a particular class of them. A class that, being a subset of the set of all models, is also more manageable and easier to study. Once that we have understood "what is going on" in these models, the extension of our comprehension to the other models of the theory should be straightforward.

---

2 Cf. (Suppes [2002], p. 54): "Roughly speaking, two models of a theory are isomorphic when they exhibit the same structure from the standpoint of the basic concepts of the theory".
2.2 *The Second Pillar: Invariance*

Representation alone is not sufficient to sustain all the weight of the philosophy of science's building. The element that is complementary to representation and that is as necessary as representation is invariance.

The concept of invariance is probably in many respects less intuitive than that of representation, but even in this case, when Suppes explains what it is used for, he starts from very commonsensical sketches.

In Suppes [2000], he gives a preliminary, very rough definition: "something is invariant if it is not varying, unalterable, unchanging, or constant." A very trivial example of something that is invariant through time is the number of my ancestors, while the number of my descendants is surely not invariant.

A concept that is closely related to the one of invariance is symmetry: more specifically, symmetry is a particular kind of invariance, namely invariance with respect to a certain transformation (like, for instance, rotation). Symmetry in geometrical figures is an example that Suppes is very keen to use, as it is precise, but at the same time very intuitive (Suppes [2000], pp. 1572-1573):

Figures symmetric under various relations are also familiar examples of symmetry, the circle above all, but, of course, the square is also symmetric under any 90° rotation. Notice the difference. The square has a very small finite group of rotations under which it is invariant, whereas the circle is invariant under an infinite group of rotations.

Already in this very intuitive setting, it is evident that the concept of invariance is a relative one: something can only be invariant with respect to something else.

In the realm of science, invariance is always defined relatively to a theory; this means that when we analyze a scientific theory, another thing that we are really interested in are those properties that are invariant with respect to some relevant transformations. More precisely, Suppes claims that what we use to call "the objective meaning" of a theory is given by the invariant properties of that theory.

This is shown for instance in the work of physicists: when they
talk about the objective description of a phenomenon, they usually refer to a description that is invariant with respect to the observer's frame of reference. As, for instance, the distance between two simultaneous spatial points, measured by different observers using clocks with the same calibration. So, this distance has objective meaning, independent of the observer.

As Suppes notes, invariance becomes particularly important when we deal with numerical quantities, because purely qualitative properties and relations are given as already invariant and then meaningful, while numerical quantities can depend on a variety of things, such as the unit of measurement. So their invariance is established up to a certain kind of transformation, and hence the importance of the pursuit of invariance in science.

In other words, invariance tells us the degree to which a representation is unique. Solving the so called "uniqueness problem" amounts to finding the set of appropriate transformations under which a representation is invariant. As an example, while classical mechanics gives a representation that is unique up to Galilean transformations, relativistic particle mechanics gives a representation unique up to Lorentz transformations.

So, the characterization of a theory is complete when we have given both a representation and a uniqueness theorem, or in Suppes' words (Suppes [1988], p. 267): "[...] a representation theorem should ordinarily be accompanied by a matching invariance theorem stating the degree to which a representation of a structure is unique."

2.3 The Architrave: Axioms

So far, I have illustrated the importance that Suppes ascribes to the notions of representation and invariance. If the task of the philosophy of science is that of analyzing the nature of science, Suppes thinks that this task is accomplished once we have shown the representation that a specific scientific theory expresses and what is invariant inside this same theory. According to Suppes, if we succeed in pointing out the representation and the invariance of a scientific structure, we elucidate and characterize such structure, thus accom-
plishing a main task of philosophy of science.

In general, the philosophy of science not only tells us which are those aspects of a scientific theory on which our analysis has to be focused, it should also tell us how we should pursue that analysis. In fact, Suppes not only encourages us to concentrate on representation and invariance, but he also tells us the methodology that in his opinion is the most appropriate for the task at hand, namely the axiomatic method (Suppes [1992], [1979]).

The axiomatic method is the core of the mathematical approach of Patrick Suppes to the problems of the philosophy of science and this is the reason why I decided to call it "the architrave" in my "architectural metaphor": his philosophy of science as a whole rests on the axiomatic method.

There are many reasons why Suppes elected this method as his favorite (Suppes [1968]). First of all, the axiomatic method constitutes a common framework for the discussion of scientific problems, and this is something absolutely not trivial. This is because, especially in the last century, science has become very specialized; the language and technicalities used in a specific discipline are most of the times absolutely obscure for people coming from different background and often hardly understandable, even for people working in different but strongly related branches of the same discipline. The axiomatic method, thanks to its abstractness and clarity, can constitute a sort of common ground on which scientists can build a minimal but necessary mutual understanding. Moreover, an explicitly axiomatic analysis of scientific structures has multiple advantages (Suppes [1968]). An evident advantage is exactly the explicitness of formalizations: by formalizing interconnected concepts, even the meaning of these concepts becomes more explicit. Another advantage, made particularly important by the aforementioned specialization in sciences, is standardization: axiomatization can create standard terminology and standard methods of conceptual analysis in

---

3 The concept of standardization, as we will see better in the following, is a very important one even in connection with the approach of formal ontologies. In particular, one of the main uses of ontology in computer science is that of making explicit the intended meaning of a terminology. So, the effort towards finding a standard in the scien-
various branches of science. Axiomatizations have a strongly general character and this allows them to go beyond the different ways in which scientific theories have been conceived, for instance by showing that different results can be overcome by finding isomorphism between antagonistic models. In other terms, two theories that give apparently different representations of the same phenomenon can be shown to have the same form with respect to a relevant property of that phenomenon (and this is where isomorphism comes into play).

Axiomatization is also a way to approach objectivity: when scientists are faced with enormous corpora of experimental data that gave birth to numerous models, normally the most precise models are isomorphic one another. A further advantage is recognizable in scientific areas where there’s still much confusion; in such areas, axiomatization is a way of isolating the base of suppositions. The selection of these suppositions would otherwise be arbitrary. Finding self-contained suppositions is a way of guaranteeing scientific objectivity. Axiomatization is also a way to analyze the minimal presuppositions necessary in order to embrace a theory. This can be done by introducing a minimal set of mutually independent and self-contained axioms. There are still a couple of points that is important to stress in Suppes’ position because they are fairly unusual.

The first one is the heuristic value that he assigns to the axiomatic method (Suppes [1983]). This is usually thought as important for the characterization of an already mature discipline. This is certainly true for Suppes, but he also thinks that the axiomatic method can be very helpful even during the periods in which disciplines are forming. The positive heuristic method applies equally well to the comprehension of scientific subjects, to the solution of specific problems, and to the formulation of new problems and this is for sure an element of novelty in Suppes’ approach. A formal analysis is heuristic when the axioms themselves facilitate the way in which we think about the subject.

The last point is the importance of this method not only for purely mathematical sciences, but also for empirical (Suppes [1974], ific practices and that aimed at explicitness present in the studies on ontology have both the goal of enhancing communication through the use of a formal method.)
[1960]) and social sciences. Firstly, Suppes rejects the idea that such a sharp distinction between pure and "applied" sciences exists; secondly, he realizes that empirical and social sciences have to deal with a great amount of experimental data and these data need to be managed in some way. Moreover, an important part in these "applied" sciences is played by measurement. Data and measurement are subjects that have been thoroughly analyzed during Suppes' entire philosophical career (Suppes [1962], [1954]) and he has found out that the axiomatic method is a very useful tool in the systematization of messy stuff, like data and measures.

3. APPROACHES BASED ON FORMAL ONTOLOGIES

3.1 Formal Ontologies: Language + Conceptualization

In recent years, an ever-growing number of researchers have begun to get interested in the study of formal ontology.

The use of the word "ontology" in computer science is considerably different from the one present in the philosophical tradition, where it indicates a branch of metaphysics, that studies existence and its main categories.

In computer science, people generally call "ontology" the conceptual schema of a given domain. These conceptual schemas are usually composed by a hierarchy of concepts related to one another through semantic relations; more elaborate ontologies are also provided with rules (like axioms or theorems) that help to better specify how the domain is structured. It is difficult to give a precise definition of ontology in the computer scientists' terms, but a definition that is fairly widely accepted is the one given by T. R. Gruber in (Gruber [1995]): "An ontology is a specification of a conceptualization".

A more detailed definition is given in (Guarino [1998], p. 4):

[...] in its most prevalent use in AI, an ontology refers to an engineering artifact, constituted by a specific vocabulary used to describe a certain reality, plus a set of explicit assumptions regarding the intended meaning of the vocabulary words. This set of assumptions has
usually the form of a first-order logical theory, where vocabulary words appear as unary or binary predicate names, respectively called concepts and relations. In the simplest case, an ontology describes a hierarchy of concepts related by subsumption relationships; in more sophisticated cases, suitable axioms are added in order to express other relationships between concepts and to constrain their intended interpretation.

Among formal ontologies, we can distinguish light-weight ontologies, that are artifacts specifically designed for applications on definite restricted domains or for very specific tasks, and foundational ontologies\(^4\), that are more general schemas, not tightly connected with particular domains, but more adaptable for general problems and thus for the communication of information. More in detail, we could say that lightweight ontologies are taxonomic structures containing primitives and composite terms and their definitions, usually built by simple relationships, as the subsumption relation\(^5\). Lightweight ontologies are normally used in well-established communities, where the intended meaning of terms is more or less already known in advance. Nevertheless, when a community evolves and crosses linguistic or cultural boarders, the need of making explicit the intended meaning of terms arises, in order to overcome ambiguities and misunderstandings. These ambiguities and misunderstandings can only be solved by elucidating the relations holding among terms and by eliciting the formal structure of the given domain. This task is accomplished by axiomatic ontologies that not only facilitate the meaning negotiation of terms, but also model the negotiation process itself.

In this paper I will focus on foundational ontologies\(^6\), as I am mainly interested in the axiomatic character of ontologies, that is certainly more prominent in the foundational case. Therefore, if an

\(^4\) Other, more accurate distinctions can be made among the different types of ontologies, like top-level vs. domain vs. task vs. application ontologies (Guarino (1998)), but such a level of accuracy is not necessary for the aims of this paper.

\(^5\) An example of subsumption relationship is: “A human is a mammal”.

\(^6\) Among axiomatic ontologies, foundational ontologies are those that address very general domains. Examples of foundational ontologies are DOLCE (http://www.isa-cnr.it/DOLCE.html) and CYC (http://www.cyc.com/cyc).
ontology is to be meant as a specification of a conceptualization, the first thing that needs to be clarified is what a conceptualization is.

Very generally, we can say that a conceptualization is a system of categories embedding a particular vision of the world or, more specifically, that it is formed by a domain and a set of relevant relations on that domain. The second thing to explain is how an ontology specifies a conceptualization. In order to understand that, we need to introduce the notion of "ontological commitment" for a language $L$.

As we know, a model for $L$ is given by a domain, the relations in this domain and a classical (extensional) function of interpretation. Analogously, $L$ has also an intensional interpretation that ranges on the domain space and its conceptual relations. Now we can say that a language commits to a certain conceptualization by means of a definite ontological commitment. All the models of the language that are compatible with the ontological commitment are called "intended models" of the language $L$ according to that specific ontological commitment.

Nevertheless, singling out the intended models of a language, relative to a particular conceptualization is only part of the deal: the absurd interpretations of the language are excluded, but still the meaning of the vocabulary terms has not been described. This is given by the ontology (Guarino [1998], p. 6):

We can now clarify the role of an ontology, considered as a set of

---

7 These are not to be intended as the "standard" extensional relations, but as intensional relations. For a more accurate account, see (Guarino [1998]).

8 The fact of having singled out the notion of intended model is not enough, as such models are in the mind of the agent who uses a certain conceptualization, unless they are exhaustively enumerated (in case they are finite in number) or, better, unless they are characterized by means of an axiomatic theory. Such axiomatic theory is what we call an ontology and it may or may not capture the intended models. At this stage, we still cannot be sure that the ontology expresses the "intensional" meaning of the terms of the language. The reason is that the choice of the primitives of the language is itself arbitrary and we cannot ascertain if the chosen primitives are exactly the ones adopted in the conceptualization. So, an ontology can be evaluated along different dimensions: coverage (it should capture as many intended models as possible), precision (it should exclude non intended models), and accuracy (it should distinguish between intended and non intended situations).
logical axioms designed to account for the intended meaning of a vocabulary. Given a language $L$ with ontological commitment $K$, an ontology for $L$ is a set of axioms designed in a way such that the set of its models approximates as best as possible the set of intended models of $L$ according to $K$.

This is well exemplified in Figure 1 below.

![Diagram]

Figure 1 – Relations between language, ontology, conceptualization and intended meaning

Thus, while a conceptualization is language-independent, an ontology depends on the language at hand.

3.2 The Bricks: Primitives

In order to conceptually structure a domain through a formal ontology, the basic elements of that domain need to be picked out. These basic elements are the so-called “primitives”, that are the most fundamental categories; all the other categories in an ontology are defined in terms of them. In model theory, primitives are the basic elements that, combined by means of logical connectives, form the structure. Their meaning thus cannot be given by an explicit definition referring to other (more basic) entities. As we will see better in a while, their definition in a sense emerges from the axioms of the ontology, that state how these primitives are related one another.
The logic underlying the ontology is neutral with respect to the choice of the primitives or the axioms.

3.3 The Mortar: Axioms

From what already said, we can think about a formal ontology as a logical theory that expresses a worldview; in a slightly more technical fashion, we could say that it is a logical theory whose models are similar to or approximate a certain conceptualization. In other words, a formal ontology is an axiomatic theory that formalizes a system of categories; this formalization entails that, even though many different interpretations of the given language are possible, the cardinality and the structure of the domain cannot vary, are univocally fixed. This means that it is possible to abstract away from a particular structure and consider the class of all the structures that are isomorphic to this one. Let’s take, for instance, a subject who’s observing a certain portion of the reality, in which she sees two blocks, one over the other and, in order to represent it, she uses the following structure:

$$U_1 = (D_1, R_1), \text{ with } D_1 = \{b_1, b_2\} \text{ and } R_1 = \{(b_1, b_2)\}$$

where $D_1$ is the domain, composed by the two blocks, $b_1$, and $b_2$, and $R_1$ is the relation of being over. Other subjects are free to assign different interpretations to the signs chosen for the representation (for instance, they can use the same signs to talk about the colors of the blocks and $R_1$ can be seen as the relation “being darker than”) but nonetheless, the cardinality and the structure of the domain are univocally fixed.

This is one of the advantages of dealing with ontologies as formal systems; other advantages are the staticity and closure of formal systems: the domain of interpretation and relations don’t change while the deductive process goes on. Thus, this process becomes a tool to make explicit the properties of the primitives and of the entities of the domain, that are already present, implicitly, in the axioms.

For all these reasons, axioms are the mortar that keeps together the bricks (the primitives) and give structure to the domain that the
ontology is to represent (Gangemi, Guarino \textit{et al.} [2001], p. 27):

By means of these formal relations we shall be able to:
- Formulate general constraints (e.g., atomicity) on all domain entities;
- Induce distinctions between entities and impose a general structure on the domain.

For instance, we can have some ground axioms in the theory describing the instantiation relation and determining which entities are particulars and which universals. Here is the example:

1. \( I(x, y) \rightarrow \neg I(y, x) \)
2. \( (I(x, y) \land I(x, z)) \rightarrow (\neg I(y, z) \land \neg I(z, y)) \)
3. \( Par(x) = \exists y(I(y, x)) \)
4. \( Uni(x) = \neg Par(x) \)

Axioms (1) and (2) describe the instantiation relation -- (1) says that it is an asymmetric relation, while (2) says it is an antitransitive relation -- then, consequently, it is possible to establish when a certain entity of the domain is a particular and when it is a universal -- definitions (3) and (4). So, after having enumerated the primitives, it is possible, thanks to these few axioms, to single out which among them are universals, which are particulars, and which particulars are instances of which universals. These and similar formal properties characterise the relations holding among the entities of the domain, thus providing a formal structure.

4. SUPPES' AXIOMATIC METHOD
AND FORMAL ONTOLOGIES: A COMPARISON

4.1 \textit{Some Differences}

While comparing the two approaches that I am describing in this paper, I will begin with pointing out some of the differences between them, since they are obviously easier to grasp.

The first important difference is in the targets and final aims of the axiomatization in the two approaches; the target of Suppes' analysis are scientific theories, while the target of formal ontologies are information systems. As a consequence, the goal of Suppes in
applying the axiomatic method is that of analysing scientific theories, while the purpose of formal ontologies is somehow more specific. Formal ontologies are mostly designed for capturing the intended meaning of the terms of a vocabulary (used to describe a certain domain). Very often a domain can be described by different vocabularies and the need of making them communicate arises\(^9\). Once that the intended meaning has been made explicit, it is much easier to match the terms of a vocabulary with those of another one, thus enhancing the communication process (Masolo, Borgo et al. [2003], p. 2):

To capture (or at least approximate) such subtle distinctions [in terminology and meaning] we need an explicit representation of the so-called ontological commitments about the meaning of terms, in order to remove terminological and conceptual ambiguities. A rigorous logical axiomatisation seems to be unavoidable in this case, as it accounts not only for the relationships between terms, but – most importantly – for the formal structure of the domain to be represented.

As an example (Oltramari, Borgo et al. [2003]), we can think about two medical information systems in which we want to look for a treatment for a disease and compare the two proposed treatments. In one of the two systems, the term \textit{treatment} is used to refer to medicaments, so physical objects, while in the other it is used to refer to therapies, so events involving of course the use of medicaments, but not only. Where the first system uses \textit{treatment}, the second uses \textit{remedy}, while where the second uses \textit{treatment}, the first uses \textit{medical therapy}. So, in order to compare the two systems, the need for a disambiguation in the use of the term \textit{treatment} is needed. To overcome the mismatch, we need to query the two systems at a more abstract level, where it is made explicit if the term \textit{treatment} refers to an object or to an event. One of the tasks of the ontology is that of making explicit the representation of these so-called intended meanings. Moreover, the ontology gives an account of the semantic connections of the meaning of \textit{treatment} in the two

\(^9\) A very instructive example is that of the web, where artificial agents providing services on the web are designed according to different criteria and thus use different vocabularies to describe resource content and capabilities.
systems, thus making explicit the commitments of the two languages to two different conceptualizations of the world (or, in this case, of the medical domain)\textsuperscript{10}.

Another relevant difference is in the role played by language in the two approaches: in his work, Suppes mostly put the emphasis on the study of models and of structures, rather than on the language used. On the other hand, the descriptive dimension (and thus the language being used) is very important for formal ontologies, as the choice of the primitives differentiates an ontology from another. Another minor difference is given by the different logical tools they use; formal ontologies (especially foundational ontologies) tend to be based on very simple logical frameworks, like first order logics, as it is very useful in communication, since it is commonly used. On the other hand, Suppes prefers to deal with set theory but, as he points out, the reason for this choice is purely pragmatic: first order logic is not sufficient when the scientific theories we are analyzing reach a certain degree of sophistication. Set theory is sufficiently expressive as to capture most of the mathematics ordinarily used in the scientific enterprise. Formal ontologies normally deal with more generic and therefore mathematically simpler domains. In both cases, the reason for the choice of a formal framework instead of another is pragmatic rather than substantial.

4.2 Some Analogies

The first analogy that I envisage in the two approaches is in the rationales for the choice of the axiomatic method. This could appear to contradict what has been said in the previous subsection, where I pointed out the different goals, as the goal is certainly an important rationale for this choice. But, even if the explicit rationale of the two approaches is different, the intrinsic one is very close.

Let's remind the advantages Suppes attributed to the axiomatic method: in particular, the fact that axiomatizations, thanks to their general character, can constitute a common ground for communications among scientists with a different background and the fact that,

\textsuperscript{10} This example is a re-elaboration from (Ottomanari (2003)), an English translation of which will appear soon on the website http://www.kva-cnr.it.
by making explicit the relations among scientific concepts, they contribute to elucidate their meanings. These desiderata are more than compatible with those of formal ontologies, i.e. building a shared setting for communication among different vocabularies and making explicit the ontological commitments in order to find the intended meaning of terms and concepts.

Coming back to the issue of the centrality of language, if it is true that the role played by language is different in the structuralist approach of Suppes and in ontologies, a parallel can still be traced. In Suppes’ view we have that a scientific structure can be axiomatized in many different ways, thus giving birth to different theories, that are comparable, by looking for isomorphism between models; in formal ontologies a conceptualization can be specified by different ontologies that are related and grouped in a library\textsuperscript{11}.

A further similarity, which is in a way a consequence of the one just mentioned, is the pluralistic attitude that both these approaches share. In Suppes’ case this pluralism is technically motivated by the multiple possible axiomatizations for the same structure but, more importantly, it is philosophically motivated by the conviction that the presence of different theoretical perspectives on the same scientific matter is a good sign, a sign of the dynamicity of a scientific discipline (Suppes [1979], p. 16): “The absence of a fundamental theory dominating a given area of science or the philosophy of science is a healthy and normal state of affairs”.

For what concerns formal ontologies, they are pluralistic in many ways: firstly, as already noted, different ontological options can be attached to a single conceptualization and different conceptualizations of the same domain can be given but, moreover, it is also possible to choose which is the most appropriate conceptualization for a definite purpose. This is due in part to the fact that ontologies are cognitive artefacts and they strongly depend on human perception, cultural imprints and social conventions. Nevertheless, in my opinion, the most important point of contact of the two approaches is a

\textsuperscript{11} Actually, the similitude is not as straightforward, since the different modules of a library are not necessarily referred to the same conceptualization. Rather, in general not only different ontologies are assumed, but also different conceptualizations.
more substantial one. Axiomatization, in formal ontology as a discipline, plays the role that Suppes recommends for scientific disciplines: it gives a representation and characterizes the invariants in the models (the single ontologies). An ontology, as a structured system of categories, gives a representation of the domain it is applied to and, at least from a mathematical standpoint, the definition of the primitives is given by the axioms: what primitives are, all their properties and relations, are written in the axioms.

CONCLUDING REMARKS

The affection of Patrick Suppes for the axiomatic method dates back to the beginning of his career as a philosopher of science, when information systems were at a very early stage of development.

It is very interesting to see how the methodology he proposed many years ago is still up-to-date and, with the due differences, applied to brand new fields of research.

It is not only the formal method applied to ontology that shows the actuality of Suppes’ approach, it is also the importance of the message contained in it for computer science in its last developments: the representation given by a particular information system is built on its invariants, on the structure that resists the changes the system goes through, for instance those induced by reasoning and other transformation processes.

To sum up, in many respects it seems possible to claim that formal ontologies show an extension of the application of the methodology proposed by Suppes for scientific structures to new subjects, namely information systems.

Laboratorio di Ontologia Applicata
Istituto di Scienze e Tecnologie della Cognizione
Consiglio Nazionale delle Ricerche
ACKNOWLEDGEMENT

I would like to thank Patrick Suppes, Claudia Arrighi, Nicola Guarino and all the other people at LOA who have kindly accepted to read the paper and to give me comments and suggestions.

REFERENCES


Roberta Ferrario

A CHI IMPORTA DELLE ASSIOMATIZZAZIONI? RAPPRESENTAZIONE, INVARIANZA E ONTOLOGIE FORMALI

Riassunto

Le due nozioni alla base della filosofia della scienza di Suppes sono quelle di rappresentazione e invarianza: nella sua visione, caratterizzare una struttura scientifica corrisponde a fornire un teorema di rappresentazione dei suoi modelli e individuare le sue invarianti.

Il teorema di rappresentazione è in un certo senso l’espressione delle scelte compiute da una teoria scientifica, mentre l’invarianza è ciò che la dota di un significato oggettivo.

Allo stesso modo, gli approcci in scienze dell’informazione basati sulle ontologie formali cercano di rendere esplicita la struttura concettuale sottostante a un certo dominio, individuandone le entità primitive, le proprietà di queste ultime e le relazioni che hanno luogo tra di esse.

Lo scopo di questo articolo è di tracciare un paragone tra l’approccio metodologico applicato da Suppes in filosofia della scienza e quello usato dagli studiosi di ontologie formali.

Come sarà mostrato, nonostante i diversi oggetti di indagine (discipline scientifiche nel caso di Suppes e sistemi informativi per le ontologie formali), a livello metodologico i due approcci presentano delle somiglianze, in particolare rispettivamente al loro appello all’assiomatizzazione.
In the first two sections of her commentary on my work, Ferrario gives an excellent summary of the views expressed in my recent book [2002], especially on the three critical concepts of axiomatization, representation, and invariance. I also like her comments in the last part of section 2, namely, section 2.3, where she stresses two points that she states about my position that are fairly unusual. The first one is the heuristic value that in earlier publications I have assigned to the axiomatic method. She is right in this because it is a common position to compare heuristics to axiomatic methods, in the sense there is no claim that heuristic methods give a full formal account of the content of a subject, or that axiomatic methods are heuristic. I also want to emphasize another point that she does. This is that I think it is important to distinguish axioms that are heuristically valuable, or put another way, are intuitive in their presentation of content. Some axioms that seem necessary and are very useful in subjects are often described as technical. This usually means they are complicated, difficult to read, and do not have a straightforward intuitive meaning. It is a desirable distinction to be made about axioms that brings the concept of heuristics within the framework of the axiomatic method itself. In my 1983 article on this subject that she cites, the main example of axiomatization I criticized on this score is Mackey’s axiomatization of quantum mechanics ([1957],

Epistemologia XXIX (2006), pp. 385-398.)
The non-heuristic character of these axioms is I think the main reason they are seldom mentioned by physicists.

The second unusual point that Ferrario mentions is my stress on the use of axiomatic methods in the empirical sciences. I think she is still right about this point, even though by now the use of the axiomatic method, in such subjects as economics, is the natural way to treat a subject in such prominent journals as *Econometrica*. She mentions a point that I have not emphasized myself enough. This is the paradoxical fact that some of the most extensive axiomatizations in use in the social sciences are to be found in the area of measurement, because of the desire to make the procedures of measurement in these new sciences completely explicit. Here the axiomatic method is used to a fare-the-well, – to give an egocentric reference – as may be found in the three volume treatise *Foundations of Measurement* ([1971], [1989], [1990]) of which I am a co-author. Put another way, the axiomatic method has very much proved itself to be of use not only in empirical sciences but also in the general methodology of the empirical sciences.

The remainder of Ferrario's paper is about formal ontologies with a use of axiomatic methods within the framework of first-order logic that has become increasingly popular in computer science. I generally agree with the comparisons she draws between the use of axiomatic methods in the sciences and in formal ontologies. The description of the use of axioms in formal ontologies in section 3.3 of Ferrario's commentary has an outlook and a viewpoint that very naturally goes back to the first modern work on the foundations of the axiomatic method, namely, the 1882 treatise on geometry of Moritz Pasch, which made explicit the formal nature of modern axiomatics, namely, that no intuitive content of any particular interpretation is made a direct part of the axioms. This methodology, well-consolidated by Hilbert in his *Foundations of Geometry* [1899], is now very much the modern view of what one means by the axiomatic method in either mathematics or the sciences. What is said in formal ontologies has direct resonance with the interesting and simple examples given by Pasch in his early work. A point that is more modern than Pasch, and clearly is of great importance in for-
mal ontologies, is the non-categorical character of the axioms (a set of axioms is categorical if and only if any two models of the axioms are isomorphic). The many successful formal axiomatizations of geometry at the end of the nineteenth century and beginning of the twentieth century were mainly aimed at categorical axioms, for example, for Euclidian, hyperbolic, and elliptic geometry. Even though, it should be noticed, non-categorical axioms were familiar in projective geometry, for example, Fano's miniature projective plane of just seven points and seven lines, the report of which was published in 1892, was just one example of the many hundreds of finite geometries that have been considered since. Such finite geometries constitute, in many ways, developments parallel to those considered in formal ontologies.

Finally, I am happy to remark that the approach to formal ontologies shares a pluralism I much advocate in the analysis of structures and theories in the empirical sciences.

REFERENCES


