Mario Valentino Bramè

DEFENDING KANT: THE AXIOMATIZATION OF PERCEPTIVE SPACE

Abstract

The axiomatic method introduced by Patrick Suppes plays a double role in saving Kant's *a priori* from some accusations based on geometry. First of all, it replies to some accusations directly coming from neo-positivistic philosophers (e.g. Carnap). Furthermore, there's another kind of help.

In defending Kant's *a priori* from certain vague accusations coming from the discovery of the *non-euclidean* geometries, some may say the teaching of Kant about geometry is to be applied to the everyday space, while the space of *non-euclidean* geometries regards to astronomical spaces.

Nevertheless, we know that also in the perceptive space there are some elements of hyperbolical and elliptical geometry.

That kind of defence of Kant's *a-priori* may become a danger for Kant himself. We may say perceptive space is the space of Kant's aesthetic (pure intuition) after imagination has made some prehensions on the "pure" indefinite space. Those prehensions (lines, planes, other simple geometrical elements) are made by means of categories in the form of postulates.

If perceptive space were exclusively euclidean, we could say the only categories permitted are those related to euclidean postulates. At an higher categorical level, of course every geometry would remain legal as a deductive system of propositions, but still there would be a kind of dangerous privilege of euclidean geometry.

To avoid this danger we should demonstrate perceptive space is not univocally axiomatizable. Suppes does it by examining some Foley's experiments.

Patrick Suppes

RESPONSE TO MARIO VALENTINO BRAMÈ

I appreciate and find very suggestive Bramè's defense of Kant's *apriori* approach to geometry. I do not think it is necessary to cite texts to agree in these remarks that Kant makes Euclidian geometry pure *apriori* synthetic, as the pure form of spatial intuition – this is not exactly the Kantian terminology but states clearly enough what he claims to have established. Unfortunately, as Bramè admits in both of the main directions of systematic application of geometry, namely, to physics and the physical world, on the one hand, and to perception and the detailed nature of perception on the other hand. There is very good evidence that the Euclidian model is not correct. Of course in the latest twists and turns of astrophysics and galaxy astronomy, the flatness of astronomical space is once again being approximated to a reasonable degree. I have something more to say on this later, but the real attack on Euclidian space in physics is at a more fundamental level of the nature of microscopic space. This includes the very space around us as we move about on earth. Again, it is not appropriate to go into the technical details, but the evidence is certainly, at the present time, overwhelming that the fiction of strong smoothness that is not only continuity but strong differentialability of ordinary Euclidian space does not hold in physics. As we approach ever smaller regimes, we do not reach zero-dimensional points but an ever more complicated environment of

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particles, generated in all likelihood, at least partly, by our efforts to make ever smaller observations.

The same point is to be made about perceptual space. The detailed experimental data, which I review in my book [2002], argues from many different directions that the space of visual perception is not in any conclusive sense Euclidian in nature. One of the arguments not given as often in the consideration of perceptual space is that the modern mathematical formulations of Euclid do not match the data. In fact, there is a conceptually clear gap between the standard geometries developed in modern mathematics since the beginning of the nineteenth century, and the kind of spaces on the other hand that are natural to modern physics or psychological models of perception. These models of perception by the way apply not only to visual space but to the other senses as well. The evidence in the case of perception is for a threshold and inside the threshold, where continuity would need to be assumed for classical geometry, the axioms required to produce such continuity, weak or strong, have no serious empirical support.

What I said about thresholds in perceptual space apply as well to thresholds in physical space. But the thresholds are on a different scale. For example, the thresholds found in physical space are orders of magnitude smaller than those in perceptual space for the instrumentation applied to the observation of physical space in experiments of a great variety are very much more refined than the thresholds so widely observed for perceptual space.

Kant's Great Merit: The Antinomies. What I've said so far sounds as if I am very much opposed to Bramè's view and the entire Kantian enterprise, but this is not how I think about the matter. I am in fact an admirer of the depth of the difficulties in modern science, or in the philosophical foundations of modern science, Kant [1781/1997] so clearly and carefully pointed out. The four antinomies: the first on the beginning of time, the second on the continuity of physical quantities or matter, the third on freedom, and the fourth on the existence of a necessary being, all present challenges, whether the thesis or antithesis is supported. Kant also recognized

that the arguments on each side were strong and that therefore there was no simple proof that one of the two sides of each antinomy was clearly the correct one. His own organization of the antinomies in such a way that he affirmed for working purposes of science, and of empirical thought more generally, the antithesis of each antinomy did not mean that he claimed to give any absolute proof of their correctness. In fact, it was exactly in the nature of his thought to see that no absolute proof was possible. His broader thinking about these matters is very clearly explained in the pages of the critique following the presentation of the four antinomies, especially in section III (A462/B490–A476/B504) of Chapter 2 of the antinomy of pure reason, which follows the presentation in section II of the four antinomies.

I do want to note that Kant affirms that the antitheses represent the position of pure empiricism, because the proofs of the theses must necessarily go beyond experience. On the other hand this doesn't mean that from experience alone the antitheses can be themselves proved. Here is his well-known assertion on this matter at A466:

> In the assertions of the antithesis, one notes a perfect uniformity in their manner of thought and complete unity in their maxims, namely a principle of pure **empiricism**, not only in the explanation of appearances in the world, but also in the dissolution of the transcendental ideas of the world-whole itself. Against this the assertions of the thesis are grounded not only on empiricism within the series of appearances but also on intellectualistic starting points, and their maxim is to that extent not simple. On the basis of their essential distinguishing mark, however, I will call them the **dogmatism** of pure reason.

Here is his firm statement two pages later on the attractions of empiricism as reflected in the antitheses:

> **On the contrary**, however, empiricism offers advantages to the speculative interests of reason, which are very attractive and far surpass any that the dogmatic teacher of the ideas of reason might promise. For with empiricism the understanding is at every time on its own proper ground, namely the field solely of possible experiences, whose laws it traces, and by means of which it can endlessly extend its secure and

> comprehensible cognition. Here it can and should exhibit its object, in itself as well as in its relations, to intuition, or at least in concepts an image for which can be clearly and distinctly laid before it in similar given intuitions. Not only is it unnecessary for the understanding to abandon this chain of natural order so as to hang onto ideas with whose objects it has no acquaintance because, as thought-entities, they can never be given; but it is not even permitted to abandon its business, and, under the pretext that this has been brought to an end, to pass over into the territory of idealizing reason and transcendent concepts, where there is no further need to make observations and to inquire according to the laws of nature, but rather only to think and invent, certain that it can never be refuted by facts of nature because it is not bound by their testimony but may go right past them, or even subordinate them to a higher viewpoint, namely that of pure reason.
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> Hence the empiricist will never allow any epoch of nature to be assumed to be the absolutely first, or any boundary of his prospect to be regarded as the uttermost in its extent, or that among the objects of nature that he can resolve through observation and mathematics and determine synthetically in intuition (the extended) there can be a transition to those which can never be exhibited *in concreto* either in sense or imagination (the simple); nor will he admit that one can take as fundamental in nature itself, a faculty (freedom) that operates independently of the laws of nature, and thereby restrict the business of the understanding, which is to trace the origin of appearances guided by necessary rules; nor, finally, will he concede that the cause of anything should be sought outside nature (an original being), for we are acquainted with nothing beyond nature, since it is nature alone that provides us with objects and instructs us as to their laws. (Kant, A468/B496–A470/B498)

It is I think absolutely important to be clear that Kant does not support empiricism as providing any absolute foundation. The virtue of the antitheses and of pure empiricism in the form that he states it is in not permitting any outlandish ideas of pure reason, unsupported by experience, to be adopted as true. It is not that on the basis of antitheses one can give a proof that the world did not have a beginning, etc. Here is a final passage I'll quote that states this very well:

> Human reason is by nature architectonic, i.e., it considers all

> cognitions as belonging to a possible system, and hence it permits only such principles as at least do not render an intended cognition incapable of standing together with others in some system or other. But the propositions of the antithesis are of a kind that they do render the completion of an edifice of cognitions entirely impossible. According to them, beyond every state of the world there is another still older one; within every part there are always still more that are divisible; before every occurrence there was always another which was in turn generated by others; and in existence in general everything is always only conditioned, and no unconditioned or first existence is to be recognized. (Kant, A474–A475/B503)

What I think can be added to Kant's wise words are the results of modern investigations, both conceptual and empirical, showing why it is a mistake to attempt to reach some absolute position and why we can give a more definitive argument, more definitive than the one given by Kant it seems to me, as to how the antinomies are to be resolved.

Modern resolution of the antinomies. Let us begin with the first antinomy to show the kind of argument I think we can now be aware of and consider, not the big bang and what may have preceded it, but the second part of the antinomy as to whether space is finite or infinite. The central point is easy to make. The measurements on which the inference that space is flat, now current in astrophysics, all involve errors in measurement and approximations. Take a closed finite space, but one that is thousands of orders of magnitude greater in diameter than the range of any of our current observations. There is no distinguishing empirically between that enormous finite space and one of infinite extent. The inference to be made from this inability to distinguish between infinite flat space and extraordinarily large finite curved space is that on the basis of experience we cannot definitely choose one over the other. Kant comes close to saying something like this, but does not quite do it because he is also concerned about the necessity of the laws of nature. This is more evident in discussing the next antinomy.

In the case of the second antinomy this same problem arises but now with the arbitrary small but discrete nature of physical quanti-

ties or matter, as opposed to its continuity. Again if we have a small enough scale, orders of magnitude below that of any equipment currently known, no distinction can be made between a very fine grid and continuous space. The measurement experiments I mentioned earlier show well enough that any program to hope for a series of ever finer experiments that asymptotically converge on the continuity of physical space is certainly a hopeless enterprise. So again, the right empirical attitude, very much in the spirit of the quotations I've given above from Kant, is that no choice between thesis and antithesis in the case of the second antinomy can be given and in each of the remaining antinomies this is the way out.

For example, the same kind of argument holds for the third antinomy, but it is worth remarking on some of the supporting argument in some detail. In Suppes [1993], I cite the beautiful results in ergodic theory that once errors of measurement, or, put another way, inevitable variability in the context of any given physical experiment is considered, it is impossible to make arbitrarily accurate measurements. Consequently, it can be shown there is no way of choosing between the purely classical deterministic theory of the idealized motion of a ball in a Sinai billiard table, that is, a billiard table with a convex obstacle in the middle, and a stochastic process version of the motion of the ball. I have outlined this on several occasions. I refer especially to the fundamental work of Ornstein and Weiss [1991]. It is worth noting that to deal with the errors, a more complicated concept of geometrical congruence is introduced to show that under this probabilistic notion of congruence, no matter how many observations are made, there is no distinguishing between a classical physics model of the motion of the billiard ball with bounded accuracy of measurement and to a Markov stochastic model.

So in the case of the third antinomy, spontaneous motion, the spontaneity of activity so brilliantly postulated in the thesis by Kant is defended, without any appeal to concepts of human or any other living activity, but to the nature of physical processes themselves. What is the story? Kant was right in pinpointing the antinomy. The modern ergodic result is that we cannot decide on the basis of expe-

rience whether the thesis or antithesis is correct. I shall omit consideration of the fourth antinomy, but the same line of argument I have just given can be followed through.

So, where I come out is that the revised neokantian position can be made quite sound on these matters, but—and this is an important *but*, what remains from my perspective is indeed the pure empiricism without any necessity of natural laws and without any *apriori* synthetic, but a methodological position that wisely requires no extension of reason beyond experience in making claims about the nature of the world (Suppes [1995]).

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