Young Children’s Comprehension of Logical Connectives

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The development of children’s understanding of logical connectives has been discussed by a number of people over the past decade and a half. Without attempting to review this literature in detail, we cite Furth and Youniss (1965), Hill (1961), Inhelder and Matalon (1960), Inhelder and Piaget (1964), McLaughlin (1963), Piaget (1957), Suppes (1965), and Youniss and Furth (1964, 1967). Nevertheless, none of these studies reports the extent to which children of preschool age show comprehension of the meaning of the logical connectives in a well-defined experimental situation.

The importance of understanding the extent and limitations of children’s mastery of the logical connectives is evident for any cognitive theory of development. The recent work in psycholinguistics, emphasizing the complex nature of the grammar and semantics of the language of children, has provided further impetus for seeking such understanding.

It seems clear that the development of a better theory about children’s behavior and the changes in that behavior with age requires much more detailed information about their linguistic habits and competence than we now have. The present study, which consists of two closely related experiments, is meant to contribute to the accumulation of such systematic information.

The data of the experiments have been analyzed in terms of several specific regression models to provide a deeper insight into what aspects of comprehension of sentential connectives are most difficult.

The formal relations between various English idioms expressing conjunction, disjunction, and negation, and the set-theoretical operations of intersection, union, and complementation are not deeply explored in this paper, but our assumptions about these connections are obvious and uncontroversial. Deeper investigation of these linguistic and semantical matters seems desirable as part of any further extensive study of children’s comprehension of logical connectives.

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EXPERIMENT 1

The primary aim of the first experiment was to investigate the extent to which children between the ages of 4 and 6 comprehend the logical connectives of conjunction, disjunction, and negation. It was also anticipated that the exact formulation of the idiom in terms of which the connectives were expressed would affect the results. Consequently, a second aim was to investigate the relative ease or difficulty of various idioms used to express the connectives. A third, subsidiary aim of the experiment was to examine the effects of sex, age, and socioeconomic status on the performances of the children in comprehending the meaning of the connectives.

Method and Procedure

Experimental design. The differences in performance as a function of the type of logical connective and idiom were examined in a within-subject design. A $2 \times 2 \times 2$ factorial design was used to examine the between-subject effects of age, sex, and socioeconomic status (SES).

Subjects. Sixty-four subjects participated in the experiment. Thirty-two kindergarten children between the ages of 5.7 and 6.7 years were drawn from two sources: a middle-class elementary school and a school in a disadvantaged area. The other 32 children were between the ages of 4.5 and 5.4 years and attended either a preschool headstart class or a middle-class nursery school. Eight boys and 8 girls were tested from each of the four sources. The children from the preschool headstart class and from the school in a disadvantaged area were considered culturally deprived by the standards of the Office of Economic Opportunity.

Experimental materials. Eighteen wooden blocks were used. Each block had two salient properties: shape (star, circle, or square) and color (red, green, or black). Each combination of color and shape was represented by two blocks. The blocks were approximately $3\frac{1}{2}$ inches square and $\frac{1}{2}$ inch deep.

Procedure. The children were pretested to ensure that they could identify the elementary properties (color and shape) of the blocks. Three children were eliminated at this stage, but were replaced by others so that 64 subjects were tested.

Each subject was tested individually. After some preliminary commands, the subjects received 12 test commands to hand various blocks to the experimenter. The subjects were told to give all the blocks asked for, and none of the others. When the subject stopped, E, who had been recording responses without looking at the $S$, turned to the $S$ and asked, "Have you finished? Have you given all the . . . (command repeated)?"

The commands were stated with as much inflection as possible. For
example, command 11 was expressed as “the things that are green-or-square,” with the hyphenated words spoken as a coherent unit. Words were stressed and pauses were used to heighten the effect of the logical connectives. Each command was repeated several times.

The commands were as follows:

1. Give me the green stars.
2. Give me the red things and the square things.
3. Give me the things that are black, but not round.
4. Give me all the red things, and then everything else, but not the stars.
5. Give me all the things that are black and square.
6. Give me the green things, or, the round things.
7. Give me the stars that are red.
8. Give me the things that are black and not square.
9. Give me all the things that are green, and then everything else but not the stars.
10. Give me the black things that are round.
11. Give me the things that are green or square.
12. Give me the things that are not round but are red.

Seven commands—1, 3, 5, 7, 8, 10, and 12—tested the conjunction or the intersection of two sets. Three of these, namely, commands 3, 8, and 12, also used negation; i.e., they asked for the intersection of sets when one of the sets was a complement. To investigate disjunction or the union of sets, four alternate forms of the commands were given. These were 2, 4, 9, and 11; of these, commands 4 and 9 involved negation or complementation. One command, 6, used the “exclusive-or” connective. The commands not using negation will be called *positive* commands; the commands using “not” will be called *negative* commands.

**Results and Discussion**

Three types of analyses were performed: one on the differences in performance between the various groups of subjects, one on the types of responses made to the different connectives and the different idioms used to express the connectives, and one on the predictive worth of a regression model.

**Group differences.** To evaluate the contribution of age, sex, and socioeconomic status (SES), five three-way analyses of variance on the number of correct responses were carried out: one each on the total score, the score to conjunction commands, to disjunction commands, to negation commands, and to the exclusive-or command. In Table 1, the significant
COMPREHENSION OF LOGICAL CONNECTIVES

TABLE 1
SUMMARY OF SIGNIFICANT VARIABLES FROM FIVE THREE-WAY ANALYSES OF VARIANCE ON CORRECT RESPONSES IN EXPERIMENT 1

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Significant variables</th>
<th>MS</th>
<th>df</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Total commands</td>
<td>SES</td>
<td>76.6</td>
<td>1</td>
<td>33.0**</td>
</tr>
<tr>
<td></td>
<td>(Error)</td>
<td>2.3</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>2. Conjunction commands</td>
<td>SES</td>
<td>34.5</td>
<td>1</td>
<td>28.3**</td>
</tr>
<tr>
<td></td>
<td>Age</td>
<td>15.1</td>
<td>1</td>
<td>12.3**</td>
</tr>
<tr>
<td></td>
<td>(Error)</td>
<td>1.2</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>3. Disjunction commands</td>
<td>SES</td>
<td>5.1</td>
<td>1</td>
<td>10.5**</td>
</tr>
<tr>
<td></td>
<td>Age × Sex × SES</td>
<td>2.3</td>
<td>1</td>
<td>4.7*</td>
</tr>
<tr>
<td></td>
<td>(Error)</td>
<td>0.5</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>4. Negation commands</td>
<td>SES</td>
<td>40.6</td>
<td>1</td>
<td>35.6**</td>
</tr>
<tr>
<td></td>
<td>Age</td>
<td>8.3</td>
<td>1</td>
<td>7.2**</td>
</tr>
<tr>
<td></td>
<td>(Error)</td>
<td>1.1</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>5. Exclusive-Or commands</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

*p < .05.

**p < .01.

results from these analyses are presented. In 4 of the 5 analyses, SES was a significant variable with children from culturally deprived homes consistently making fewer correct responses than children from advantaged homes. Age also was important in comprehending the connectives of conjunction and negation, with the older children making more correct responses than the younger children. However, it should be noted that the connectives of conjunction and negation were not independent. Sex did not affect performance differentially. Finally, only 1 of a possible 20 interactions was significant. It tentatively may be concluded that the 3 main effects are independent of one another. The fact that socioeconomic status was a uniformly more significant variable than age is to be emphasized.

Response distributions. The number of correct responses to a logical connective is an estimate of the difficulty of the operation. The rank-order of the difficulty of the binary connectives from least to most difficult is as follows: conjunction (71% correct), exclusive-or (67%), and disjunction (11%). Significant differences are found between conjunction and disjunction (z = 9.1, p < .001), between exclusive-or and disjunction (z = 8.2, p < .001), but not between conjunction and exclusive-or. For the combined connectives negation substantially increases the difficulty of the commands (z = 3.8, p < .01). However, it does not affect the rank-order of the different connectives, and from least to most difficult the order
is as follows: positive conjunction (81% correct), exclusive-or (67%), conjunction-negation (56%), positive disjunction (18%), and disjunction-negation (6%).

The errors that the children made to the commands indicate the source of difficulty in understanding connectives. In Table 2 the notation used to describe the responses is presented. In Table 3 the response distribution for the 12 commands is shown. For the positive conjunction commands, 3 of the 4 commands (1, 7, and 10) have similar distributions. The responses to the three conjunction-negative commands (3, 8, and 12) also show a similar distribution to one another. The distribution of the responses to the two positive disjunction commands (2 and 11) are not alike, a difference probably due to the different idioms used.

In general, an inverse relationship exists between number of blocks for a correct response and performance.

When the command was for the set $X \cap \bar{Y}$, giving the intersection of

\begin{table}
\centering
\begin{tabular}{|l|l|l|}
\hline
Symbol & Definition & Example: $X = \text{red}$, $Y = \text{square}$
\hline
$X$ & The set of elements with attribute $X$. & All the red blocks.
$X \cap Y$ & Conjunction. The intersection of sets $X$ and $Y$. Each object has both attributes $X$ and $Y$. & The red squares.
$X \cup Y$ & Disjunction. The union of sets $X$ and $Y$. Each object has at least one of the attributes $X$ and $Y$. & The red blocks and the square blocks.
$X$ or $Y$ & The exclusive-or (All members of the set $X$ or all members of the set $Y$, but not both). & The set of red blocks or the set of square blocks.
$\bar{X}$ & Negation. The complementary set of $X$. The not-$X$ objects. & Green blocks and black blocks.
$(1/n)X$ & The incomplete set of $X$ objects. & Five or less (of the six) red blocks.
$(1/n)(X \cup Y)$ & The incomplete set of blocks belonging to the union of $X$ and $Y$, where members of both $X$ and $Y$ are represented. & Five or less of the red and five or less of the square blocks.
$(1/n)\bar{X}$ & The incomplete complementary set where members of both subsets in the complementary set are represented. & Some, but not all of the green blocks and black blocks.
Misc. & Miscellaneous—any response not defined by the above categories. &
\hline
\end{tabular}
\caption{Legend of Notation Used to Describe Subjects' Responses in Both Experiments}
\end{table}
one set with only a part of the complementary set was the most frequent error. It is not clear whether the difficulty was in identifying the extension of the complementary set or in the operation of intersection. Some recent evidence (Feldman, in press) indicates that 4-6-year-olds have difficulty in being exhaustive with the complementary set, which seems to suggest that the complementation caused the difficulty in these commands rather than the intersection. Another frequent error was to give the first-mentioned set, as may be seen in Table 3.

The negative disjunction commands (4 and 9) were difficult for the subjects, and many errors were made. The first-mentioned set appears in 9% of the responses. Observations of the children revealed that many of them had genuine conflict over where to place the blocks that belonged to \( X \cap \bar{Y} \); for example, in command 4, placement of the red stars when the red things and the not-stars were requested. Many children first included the red stars (in response to command 4) with the objects they gave to the experimenter, but then verbalized “But these are stars, and you said the red things and the things that are not stars,” and with this comment removed the red stars. Although an order of selection

### Table 3

**Distribution of Responses to the 12 Commands of Experiment 1**  
(Data expressed as percentages)

<table>
<thead>
<tr>
<th>Connective</th>
<th>Command</th>
<th>( X \cap Y )</th>
<th>( X \cup Y )</th>
<th>( X )</th>
<th>( \bar{Y} )</th>
<th>Misc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X \cap Y )</td>
<td>1</td>
<td>97*</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>42*</td>
<td>13</td>
<td>22</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>96*</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>92*</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>( X \cup Y )</td>
<td>2</td>
<td>11*</td>
<td>33</td>
<td>33</td>
<td>0</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>47*</td>
<td>3</td>
<td>25</td>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>( X \lor Y )</td>
<td>6</td>
<td>9</td>
<td>6</td>
<td>45*</td>
<td>23*</td>
<td>17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connective</th>
<th>Command</th>
<th>( X \cap \bar{Y} )</th>
<th>( X \cup \bar{Y} )</th>
<th>( X \cap (1/n) \bar{Y} )</th>
<th>( X \cap Y )</th>
<th>( X )</th>
<th>( \bar{Y} )</th>
<th>Misc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X \cap \bar{Y} )</td>
<td>3</td>
<td>56*</td>
<td>0</td>
<td>23</td>
<td>2</td>
<td>14</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>56*</td>
<td>0</td>
<td>25</td>
<td>3</td>
<td>8</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>56*</td>
<td>0</td>
<td>30</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>( X \cup \bar{Y} )</td>
<td>4</td>
<td>30*</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>12</td>
<td>43</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>42</td>
<td>3*</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>38</td>
<td>6</td>
</tr>
</tbody>
</table>

* Correct response.
clearly was suggested by the phrase “and then everything else . . .” very few children selected their blocks in this manner. Most children picked up each block as it came to hand and apparently tested it against a memorized version of the command.

For the exclusive-or command the correct response category is probably inflated, for the first-mentioned set is a highly probable component of the response irrespective of the connective used.

Idiom. Table 3 shows that the form of the idiom used to express a particular logical connective affected the difficulty of the command for the subjects. In the case of conjunction, the idiom of command 5, “Give me all the things that are X and Y,” was especially difficult. The other three (commands 1, 7, and 10) were quite easy, as reflected in the high proportion of correct responses. As for disjunction, only one idiom was understood with any success and that was “Give me the X things and the Y things” (command 2). The idiom of command 11 was obviously very difficult for the children.

Regression models. The discussion of regression models follows Suppes, Hyman, and Jerman (1967). The main task is to identify the factors that contribute to the difficulty of the commands. Factors to be examined include variation in the connective, the idiom, and the order of the properties. As a matter of notation, the jth factor of command i in a given set of commands is denoted by $f_{ij}$. The statistical parameters to be estimated from the data are the weights $a_j$ attached to each factor. We emphasize that the factors identified and used here are not abstract constructions from the data. Rather, they are always objective factors identifiable by the experimenter in the commands themselves, independent of any data analysis. Which factors turn out to be important is a matter of the estimated weights $a_j$. Let $p_i$ be the observed proportion of correct responses to command i for the group of subjects. The central task of the model is to predict these observed proportions. The natural linear regression model in terms of the factors $f_{ij}$ and the weights $a_j$ is simply

$$p_i = \sum_j a_j f_{ij} + a_0.$$ 

All the factors $f_{ij}$ used are 0,1-variables that take the value 1 when present and 0 when absent. The 4 connective factors were $C_1 =$ disjunction, $C_2 =$ conjunction, $C_3 =$ negation, and $C_4 =$ exclusive-or. The second type of factor considered was the form of the idiom used. Four idiom factors were used, namely, $I_1 =$ Give me the things that are X . . . , $I_2 =$ Give me all the X things and everything else . . . , $I_3 =$ Give me the X things and/or the Y things . . . , and $I_4 =$ Give me the X (things) that are Y. This classification included 11 of the 12 commands. The command not
included was the first one, for it did not fit any of the 4 categories $I_i$. The final factor used was an order variable, $D_i$, which took the value 1 when shape was the first-mentioned dimension.

A standard stepwise, multiple linear regression program was used to obtain regression coefficients, multiple correlation $R$, and $R^2$. For the regression equation

$$p_i = 0.89 - 0.31C_1 + 0.08C_2 + 0.13C_3 - 0.55I_1 - 0.65I_2 - 0.25I_3 - 0.05I_4 + 0.03D$$

the multiple $R$ is 0.995, with a standard error of 0.0203. Figure 1 shows the predicted and observed success ratios. Although the fit is good, it must be remembered that 9 parameters—the regression coefficients—are being estimated (and thus 8 structural variables are being used) to make 11 predictions.

If we reduce the number of variables in the regression equation, the problem of interpreting the coefficients is made easier and the reduction in multiple $R$ and $R^2$ is not very great. Considering only the first 4 variables that entered in the stepwise regression, the equation becomes

$$p_i = 0.64 - 0.26C_1 + 0.29C_2 - 0.39I_1 - 0.32I_2$$

with a multiple $R$ of 0.991, and a standard error of estimate of 0.0579.

Several features of the regression coefficients should be noted. Disjunction commands are difficult, and conjunction commands are easy. Negation does not enter into this regression equation, and the predictions are satisfactory without this variable. Figure 2 shows the predicted and observed success ratios.

![Graph showing predicted and observed success ratios](image-url)

**Fig. 1.** Predicted and observed success ratios with 8 variables in the regression equation, Experiment 1.
FIG. 2. Predicted and observed success ratios with 4 variables in the regression equation, Experiment 1.

EXPERIMENT 2

Both regression analyses in Experiment 1 show the significance of connective and idiom variables. In order to investigate further the role of idioms in children's understanding of sentential connectives, we performed a second experiment with a new group of subjects in which the connectives and idioms were standardized in a manner described below.

Method and Procedure

Experimental design. The subjects were divided into 4 groups, with age and sex equated across the groups. Each group was given the same set of 12 commands, with the order of the commands different for each group. Thus, type of connective and type of idiom were within-subject variables and order of commands was a between-subject variable. Each subject was tested individually.

Subjects. The 112 subjects between 4.6 and 6.0 years of age were drawn from the Stanford Nursery School and from the kindergarten classes of local elementary schools.

Experimental materials. These were the same as in Experiment 1.

Procedure. The task and the instructions were similar to those described in Experiment 1. However, variation in idioms and connectives was reduced. Three forms of idioms were used. They were:

1. Give me the things that are X and/or Y.
2. Give me the X things and/or the Y things.
3. Give me the X and/or Y things.

The operations included 6 disjunction commands, 4 conjunction com-
mands, and 2 exclusive-or commands. Half of the commands within each connective-type involved negation.

The commands were as follows:

1. Give me the things that are red and square.
2. Give me the round and black things.
3. Give me the things that are round or green.
4. Give me the black things and the square things.
5. Give me the red or star things.
6. Give me the stars or the green things.
7. Give me the things that are stars and not black.
8. Give me the red and not round things.
9. Give me the things that are red and not square.
10. Give me the round things and the not green things.
11. Give me the square or not green things.
12. Give me the black things or the not star things.

Four different orders of the commands were given.

Results and Discussion

Response distributions. An analysis of the responses made to the 12 commands is presented in Table 4. The most striking finding for the 6 positive commands is that irrespective of the connective and the idiom used the distribution of responses for different commands is similar; the most frequent response was the intersection of 2 sets, followed in frequency by one of the mentioned sets. However, the connective influenced the responses, for in 3 of the 6 commands the most frequent response was the correct one. Furthermore, an examination of the intersection response over all 6 positive commands revealed that the most frequent intersection response was to an intersection connective. Similarly, the most frequent union response was to a union connective.

The responses to the commands using negation showed the same general trend, with similar distributions of responses even though different idioms and different connectives were used. Unlike the responses to the positive commands where the first- and second-mentioned sets were given with approximately the same frequency, for the negative commands, the first-mentioned set was given significantly more frequently than the second-mentioned set ($z = 6.6$, $p < .01$). However, since the second-mentioned set was always the complementary set, it is likely that the preference was less a primacy effect than an avoidance of the complementary set.

To test sequence effects, chi-square tests were performed on each of the 12 commands over the 4 different orders of administering the com-
TABLE 4
DISTRIBUTION OF RESPONSES TO THE 12 COMMANDS OF EXPERIMENT 2
(DATA EXPRESSED AS PERCENTAGES)

<table>
<thead>
<tr>
<th>Connective Command</th>
<th>X ∩ Y</th>
<th>X ∪ Y</th>
<th>(1/n)</th>
<th>(1/n)</th>
<th>(1/n)</th>
<th>(1/n)</th>
<th>All</th>
<th>Misc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>X ∩ Y</td>
<td>1</td>
<td>42</td>
<td>10</td>
<td>5</td>
<td>7</td>
<td>1</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>34</td>
<td>10</td>
<td>9</td>
<td>13</td>
<td>0</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>30</td>
<td>7</td>
<td>16</td>
<td>17</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>X ∪ Y</td>
<td>4</td>
<td>21</td>
<td>14</td>
<td>21</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>34</td>
<td>6</td>
<td>19</td>
<td>17</td>
<td>0</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>X or Y</td>
<td>6</td>
<td>21</td>
<td>6</td>
<td>21</td>
<td>13</td>
<td>0</td>
<td>4</td>
<td>13</td>
</tr>
</tbody>
</table>

Regression models. The four regression models tested were these: a connective model with conjunction, disjunction, negation, and exclusive-or as the variables; an idiom model with one variable corresponding to each of the idioms used; a connective-idiom model with all the idioms and all the connectives of the first two models included; and a connective-interaction model with variables of conjunction, disjunction, exclusive-or, conjunction-negation, disjunction-negation, and exclusive-or-negation. The results of these 4 models are summarized in Table 5. The column headed Number of Variables gives the number of variables that entered the stepwise regression with significant effect.

Although the connective-interaction model had the greatest predictive power, it used 5 variables to predict 12 items. For a very small reduction in predictive power, we may use a two-variable connective model, the variables being disjunction and negation. The results for this connective...
TABLE 5
SUMMARY OF FOUR REGRESSION MODELS BUILT TO PREDICT CORRECT RESPONSE TO 12 COMMANDS

<table>
<thead>
<tr>
<th>Model</th>
<th>$R$</th>
<th>$\sigma$</th>
<th>$R^2$</th>
<th>Number of variables*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connective</td>
<td>0.931</td>
<td>0.354</td>
<td>0.867</td>
<td>2</td>
</tr>
<tr>
<td>Idiom</td>
<td>0.067</td>
<td>0.918</td>
<td>0.005</td>
<td>1</td>
</tr>
<tr>
<td>Connective-Idiom</td>
<td>0.936</td>
<td>0.417</td>
<td>0.877</td>
<td>5</td>
</tr>
<tr>
<td>Connective Interaction</td>
<td>0.992</td>
<td>0.154</td>
<td>0.983</td>
<td>5</td>
</tr>
</tbody>
</table>

* This refers to the number of variables which entered into the model with significant effect.

The two-variable model is shown in the first line of Table 5. Note that $R^2$ is 0.867 for this two-variable model, which represents a surprisingly good fit for a model with only 3 parameters and contrasts sharply with the very bad fit of the idiom model. In the idiom case, only one variable entered the stepwise regression; the remaining variables did not significantly improve the fit. Although $R^2$ is 0.867 for the connective model, discrepancies still existed between the predicted and observed probabilities as shown in Table 6.2

TABLE 6
OBSERVED AND PREDICTED PROBABILITIES OF AN ERROR USING THE TWO-VARIABLE CONNECTIVE MODEL, EXPERIMENT 2

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.58</td>
<td>0.47</td>
<td>7</td>
<td>0.66</td>
<td>0.83</td>
</tr>
<tr>
<td>2</td>
<td>0.66</td>
<td>0.47</td>
<td>8</td>
<td>0.74</td>
<td>0.83</td>
</tr>
<tr>
<td>3</td>
<td>0.93</td>
<td>0.95</td>
<td>9</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>4</td>
<td>0.58</td>
<td>0.95</td>
<td>10</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>5</td>
<td>0.94</td>
<td>0.95</td>
<td>11</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>6</td>
<td>0.66</td>
<td>0.47</td>
<td>12</td>
<td>0.73</td>
<td>0.83</td>
</tr>
</tbody>
</table>

CONCLUSIONS

The two experiments reported here lead to results that must be regarded as preliminary in character. More extensive and more detailed studies, especially of the language in which logical connectives are embedded, are required before any general conclusions about the comprehension of logical connectives.

To avoid problems about the conservation of probability, i.e., to guarantee the predicted $p_i$ always lie between 0 and 1, in Experiment 2 the standard transformation

$$ z_i = \log[(1 - p_i)/p_i] $$

was used, and the regression analysis was made in terms of $z_i$, not $p_i$. The reported $R^2$ is for $z_i$. 
hension of logical connectives can be drawn. On the other hand, the results of the 2 experiments, especially the results embodied in the several linear regression models presented in the paper, show that we can account for a large part of the variance in responses of the children by looking at the particular connectives used in a command and also by examining the idiom in which these connectives were expressed. When the idioms are standardized as in the second experiment, the analysis of the connectives alone is sufficient to account for more than 85% of the variance in the behavior. It is to be noted of course that this remark applies to the mean probabilities of response, not to individual responses. Considering the results that have been reported in a wide variety of literature, it is not surprising that negation enters as an important variable in the second experiment. From the studies of Eifermann (1961), Feldman (in press), Wason (1959, 1961), Wason and Jones (1963), and others, it might have been predicted that in a regression analysis of comprehension, negation would be a salient connective in terms of difficulty of comprehension. On the other hand, it was unexpected that negation would not be a significant variable in the first experiment. This is probably the result of a considerable dependence between negation and idiom variables, with the idiom variables being somewhat better predictors.

It is easy enough to characterize additional lines of research needed in terms of the comprehension of logical connectives. We have not been able to present in this paper any picture of the developmental sequence in terms of age. It would be desirable to know more about how comprehension changes with age and with linguistic exposure, and also to have a deeper understanding of why particular sorts of errors were made.

REFERENCES


COMPREHENSION OF LOGICAL CONNECTIVES


