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Sets and numbers in grade one, 1959–60*

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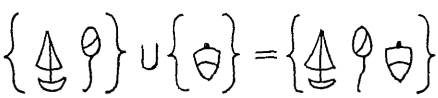
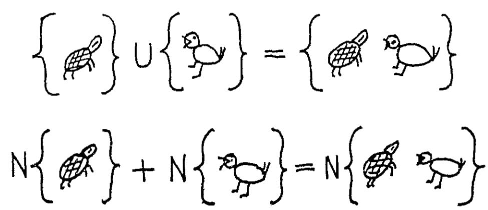
There are at least two major reasons for beginning first-grade arithmetic with the explicit introduction of the notion of set and appropriate notation for sets and operations upon them. In the first place, sets are concrete objects. Numbers are not concrete objects. Operations upon sets are more meaningful to the student than operations on numbers. The putting together of sets of physical objects, for instance, is a concrete operation. The addition of numbers is not an operation on physical objects. This is illustrated in the following example of the union of two sets.



The many exercises in the grouping of objects displayed in current workbooks are based on this idea of the union of two or more sets.

In the second place, the prior introduction of sets permits a simple and precise characterization of numbers as properties of sets. The student learns that a number is a property of a set. Continual emphasis on this definition of numbers precludes learning number symbols as meaningless symbols or addition as a meaningless operation. Numbers are properties of sets. The operation of addition of numbers is simply a general way of combining families of sets of things without paying any real attention to the things themselves. The introduction of notation for sets permits consideration of the operations of arithmetic without commitment to the particular notation of Arabic numerals. The possibility of breaking away from the reiterated use of this single notation does much to eliminate the tendency to focus on the numerals themselves without consideration of their meaning. To illustrate these points about concreteness and meaningfulness the introduction of addition at three levels is indicated below. In the first line the concrete operation of putting sets together, that is, of forming their union, is indicated. In the second line the addition of numbers is indicated; the notation of putting a large *N* before a set indicates the number of elements in the set. In the third line Arabic numerals are introduced.



$$1 + 1 = 2$$

The second line of this example illustrates the fact that addition is a general way of combining sets of objects without paying attention to the objects themselves.

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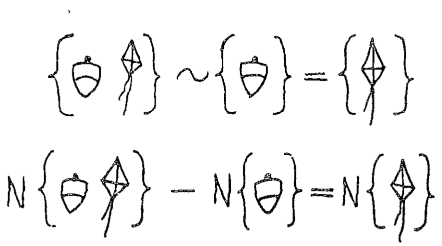
Experimental classes

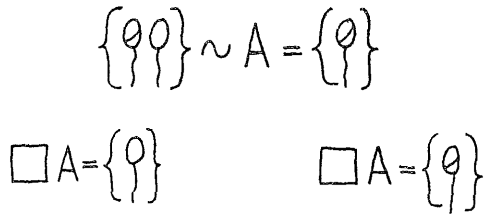
From January to June, 1960, the ideas described above were tried on four first-grade classes in the Public School System of Palo Alto, California. The ideas were embodied in a workbook, *Sets and Variables, Book I*, written by the first author. The order of development of ideas is most easily indicated by a brief description of the chapters of the workbook.

Chapter I is concerned with the notion of sets itself and the identity of sets. The exercises in this chapter, as in all others, consist primarily of the student choosing one of several possible answers and indicating his choice by checking the appropriate answer box. Chapter II is concerned with the introduction of the numbers one through six as properties of sets. Drill on recognizing and using Arabic numerals is emphasized in this chapter. Chapter III deals with the union of sets, with the restriction that the combined set contain not more than six members. Chapter IV introduces addition of numbers in the manner indicated above; that is, by first considering the union of sets, then the addition of numbers with reference to sets of concrete objects, and finally by use of the Arabic numerals. Chapter V considers the difference of sets containing up to six objects, and Chapter VI contains a development of subtraction of numbers corresponding to that of addition. The three conceptual levels of the development of subtraction are indicated in the following example.



$$2 - 1 = 1$$

In Chapter VII the capital letters A, B, C, D, E, F, and G are introduced as variables to stand for sets and the lower-case letters x, y, and z are used to represent numbers. In the first case the students are asked to solve problems of the following type. (A check is to be placed in the box associated with the correct answer.)



In the second case variables x, y, and z are used in equations which are simple enough so as not to require algebraic methods of solution. For example,

$$x+3=5.$$

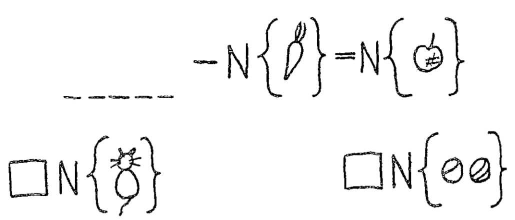
☐ $x=3$ ☐ $x=1$ ☑ $x=2$

Finally, in Chapter VIII, the empty set is introduced; in the present workbook a capital lambda (Λ) is used to indicate the empty set. The revised version will use the notation $\{\ \ \}$. Zero is introduced as a property of the empty set. Thus,

$$N\{\ \ \}=0.$$

(These eight chapters are comprised of 199 pages of exercises.)

Exercises of the following character are included in the workbook.



The problem is to decide what size set is needed to fill in the blank. Students who are facile in the rote operations of addition and subtraction often need considerable drill on exercises of this character.

There were 81 children in the four second-semester, first-grade classes who used the workbook experimentally. The classes were selected on a voluntary basis

Table 1
Summary of percentage of errors—Group A

	Part I	*Part II*	*Part III*	*Part IV*	*Part V*	*Total*
No. of errors ($N=37$)	100	131	93	127	27	478
Per cent incorrect	27.0	19.7	50.3	28.6	6.9	25.8

Table 2
Summary of percentage of errors—Group B

	Part I	*Part II*	*Part III*	*Part IV*	*Part V*	*Total*
No. of errors ($N=35$)	81	65	90	121	10	367
Per cent incorrect	23.1	10.3	51.4	28.8	5.7	21.0

Monthly meetings were held with the four teachers using the workbook, at which times the teachers were asked to comment critically on the material that had been used. As a result of these criticisms the workbook was revised in a number of respects for continued experimental use during the academic year 1960–61.

Compared with the current workbooks for first-grade arithmetic the present program introduces an extensive mathematical vocabulary and notation. It is therefore of particular interest to note that, without exception, the teachers reported that their students were having no difficulty with the set-theoretical notation. They stated that the major part of the workbook would in fact be appropriate for a first-semester first-grade class, the level for which the workbook is intended. As long as the notation introduced is explicit and precise and corresponds to simple concepts, no difficulties of comprehension seem to arise.

Results

At the end of May, 1960, three of the experimental classes had completed the workbook, and the fourth class had covered all the material except for that dealing with set variables (A, B, etc.) and numerical variables (x, y, and z). The following testing procedure was devised. Two of the classes that had finished the workbook were tested on an Atronic Tutor, a teaching device; the remaining two classes were given a written test. Both groups were given the same 50 questions, covering all the material in the workbook. Henceforth the classes tested on the teaching device will be referred to as Group A, and the classes tested with a written test will be referred to as Group B.

The Atronic Tutor, which was used to test Group A, was devised by General Atronics Corporation and is at present being used experimentally. The student is shown a problem on a white card under a plexiglas window. Underneath the problem on the same card there appear from two to four possible answers. Four keys are built into the machine in front of the plexiglas window. The student makes his response by pushing the key corresponding to the answer that he believes to be the correct one. Additional cards are retained under a panel above the immediate problem, also under the plexiglas window. If the student pushes the key corresponding to the correct answer, the next card drops down. If he makes an incorrect response, nothing happens; he has to continue until he pushes the key corresponding to the correct answer. For the purposes of this experiment, 50 cards were made up, each one containing one problem with three possible answers, covering all the material in the workbook. Each of the four answer keys was covered with one of the following colors: blue, black, red, yellow. Beside each of the three possible answers on each card was placed a dot of one of the four colors covering the answer keys. The order of correct responses was randomized. Each of the students in Group A was tested individually. The machine contains a dial which records the total number of responses made by each student, but does not record any information on which specific problems were missed or on how many incorrect responses were made on the problems which were missed. Therefore the tester observed each student while he was taking the test and recorded his performance on each of the 50 problems. The amount of time required by each student to complete the 50 problems ranged from 10 to 55 minutes.

Group B was given a written test containing exactly the same problems given to Group A. An answer box was placed beside each of the three possible answers to each problem. The two classes comprising Group B were tested separately; the members of each class took the test simultaneously. The length of time needed by each student to complete the written test ranged from 15 minutes to one hour.

For the purpose of evaluating the results, the 50-question test was divided into the following five parts: Part I, Identity, union and difference of sets, 10 questions; Part II, *N* operation, addition and subtraction of numbers, 18 questions; Part III, Set variables, 5 questions; Part IV, Numerical variables, 12 questions; Part V, Empty set and zero, 5 questions.

Tables 1 and 2 indicate the number of errors made in each group. For the purpose of comparison, only the first responses were counted in tabulating the results obtained from Group A. Clearly the most difficult part of the test for both groups was Part III, Set variables.

As it was suspected during the Group A testing procedure that the subjects were having difficulty adjusting to the teaching device, it was decided to compare the results obtained from the two groups. Because one of the classes of Group B had not covered the material in Parts III and IV, the results obtained on these two groups of questions were eliminated. Only Parts I, II and V were considered. A t-test was applied to total scores on these groups. The result was significant at the .05 level. The mean number of errors per student in Group A was 6.95, and for Group B was 4.4, where the total number of items was 33. The students who took the written test seem to have been significantly more successful than those who were tested on the Atronic Tutor. This fact can perhaps be explained by the previously mentioned observation that the subjects in Group A appeared to be having some difficulties in adjusting to the machine, which was new to them, whereas the students in Group B were presented with a test identical in format with the workbook.

Clearly the most difficult part of the test for the students was that which covered the use of the set variables (Part III). It is significant to note that there was most success on Part V, the empty set and zero. The concept of zero when presented in a clear and concise fashion is easily grasped by first-grade students. The greater difficulty of Part I in comparison with Part II is perhaps to be explained by the fact that the children had had considerable training in number concepts and none in set concepts before using the book.

On the basis of these results the experimental arithmetic curriculum for 1960–61 has been changed in two major respects. The use of set variables has been dropped, and the empty set and zero have been introduced much earlier.